# Optical metamaterials

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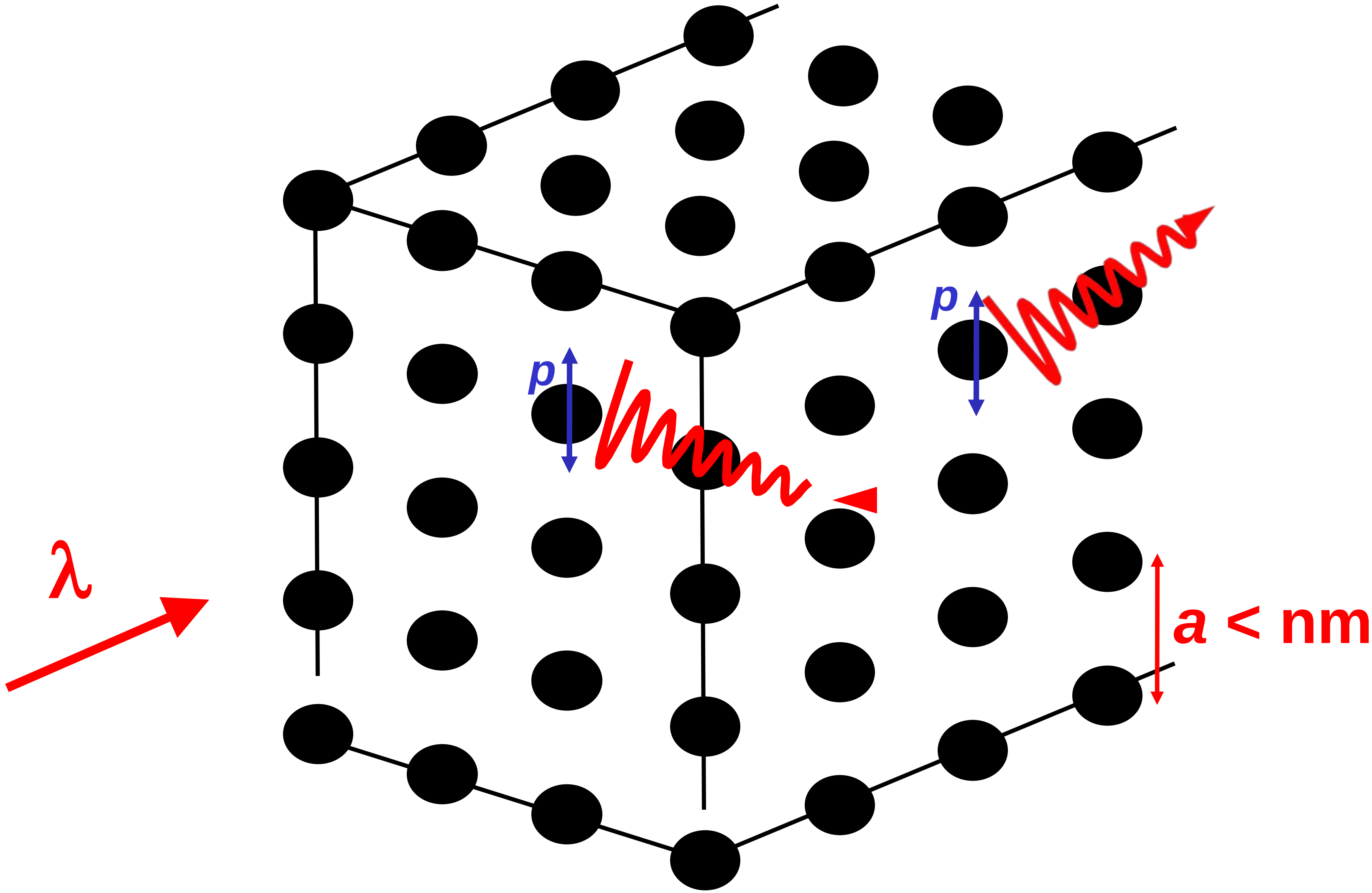
# Google search image: metamaterial


« Le passe-muraille » (1950 Jean Boyer)

Complex  nanostructure with subwvelength elementary cells
-fabricated with advanced technologies tools
-designed with advanced numerical tools

$\rightarrow$ New concepts – new physics
$\rightarrow$ New devices (sometimes)

# Back to the basis: real materials

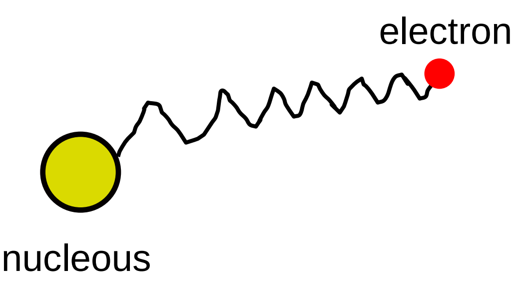


# dielectric permittivity

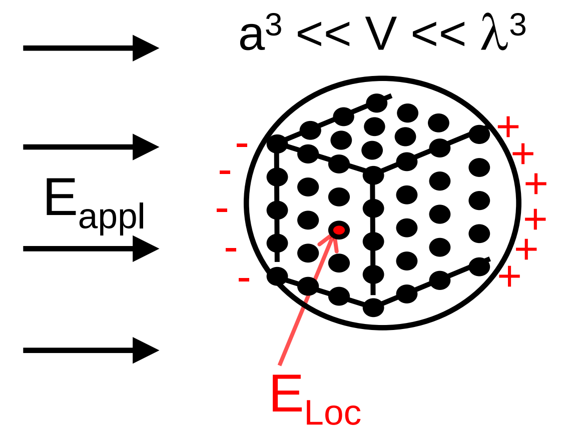
Lorentz oscilator model

$$m\ddot{x}+f\dot{x}+kx = -eE_{Loc}\exp(-i\omega t)$$

$$p(\omega) = -ex(\omega) = \varepsilon_0\alpha(\omega)E_{Loc}(\omega)$$




$$\alpha(\omega) = \frac{e^2}{m\varepsilon_0}\,\frac{1}{\omega_0{}^2-\omega^2-i\gamma\omega}$$ (polarisability)

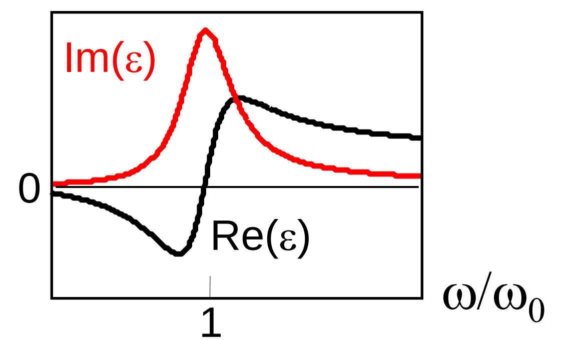


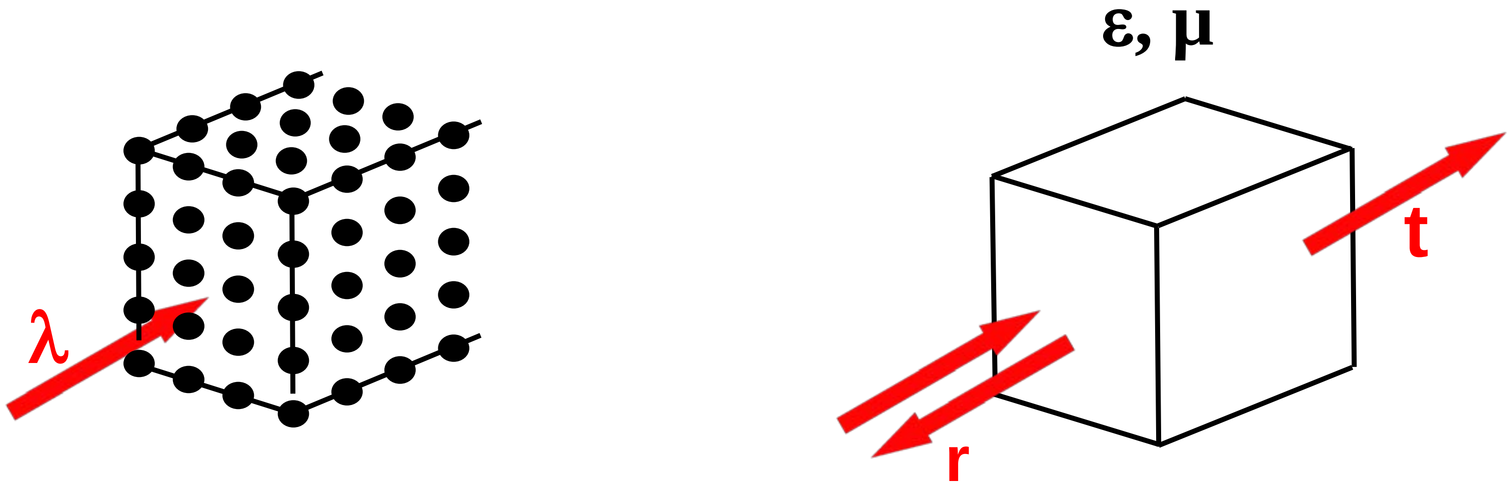

Clausius-Mosotti relation

Relation between $E_{Loc}$ and $E_{appl}$

shape of the ellipsoid? (depolarization factor)

Dielectric constant: $\varepsilon_r(\omega) = 1+\chi(\omega)$




λ
ε, μ
r
t

for linear, local,
causal materials

$$\nabla \times \mathbf{E} = -i\omega\, \mu_0\, \mu\, \mathbf{H}$$

$$\nabla \times \mathbf{H} = i\omega\, \varepsilon_0\, \varepsilon\, \mathbf{E}$$

material property after replacement of a complex heterogeneous medium by a uniform medium (homogeneisation) for linear, local, causal materials

$$\nabla\times\mathbf{E} = -i\omega\,\mu_0\,\mu\,\mathbf{H}$$

$$\nabla\times\mathbf{H} = i\omega\,\varepsilon_0\,\varepsilon\,\mathbf{E}$$

$$\Downarrow$$

$$\mathbf{E} = \boldsymbol{E_0}\, exp(i\, n\, \omega/c\, z)$$

$$n = sqrt(\varepsilon\mu)$$

wave property

material property

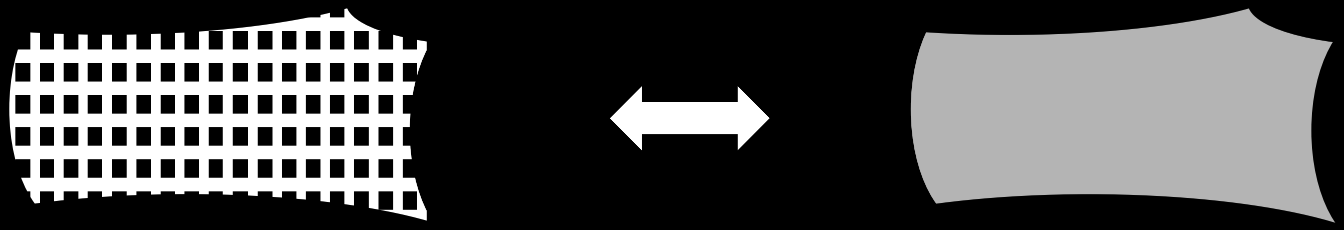

Two distinct approach:

1. Metamaterial approach ($a \ll \lambda \rightarrow$ mesoscopic scale for averaging exists): $\varepsilon_{eff}$ and $\mu_{eff}$.

2. Bloch mode approach, ($a \ll \lambda \rightarrow$ only a single Bloch mode propagates): $n_{eff}$

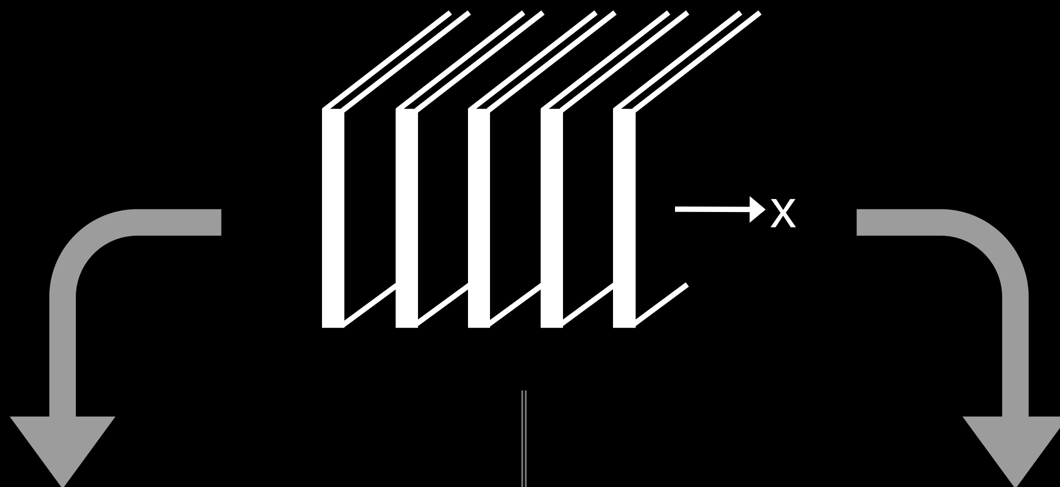
$n_{eff} = \mathrm{sqrt}(\varepsilon_{eff}\, \mu_{eff})$ ?

# Main homogeneisation result (static limit $a/\lambda \to 0$)



for $a/\lambda \to 0$, the composite medium becomes strictly equivalent to an uniform anisotropic medium and there is no magneto-optic coupling

# Example: 1D periodic systems
# (static limit $a/\lambda \to 0$)




$$\varepsilon_{eff} = \begin{bmatrix} \langle 1/\varepsilon \rangle^{-1} & 0 & 0 \\ 0 & \langle \varepsilon \rangle & 0 \\ 0 & 0 & \langle \varepsilon \rangle \end{bmatrix}$$

Uni-axial material

ordinary wave

$$\frac{|k|^2}{\langle \varepsilon \rangle} = (\omega/c)^2$$

extraordinary wave

$$\frac{(k_x)^2}{\langle \varepsilon \rangle} + \frac{(k_y)^2}{\langle 1/\varepsilon \rangle^{-1}} + \frac{(k_z)^2}{\langle 1/\varepsilon \rangle^{-1}} = (\omega/c)^2$$

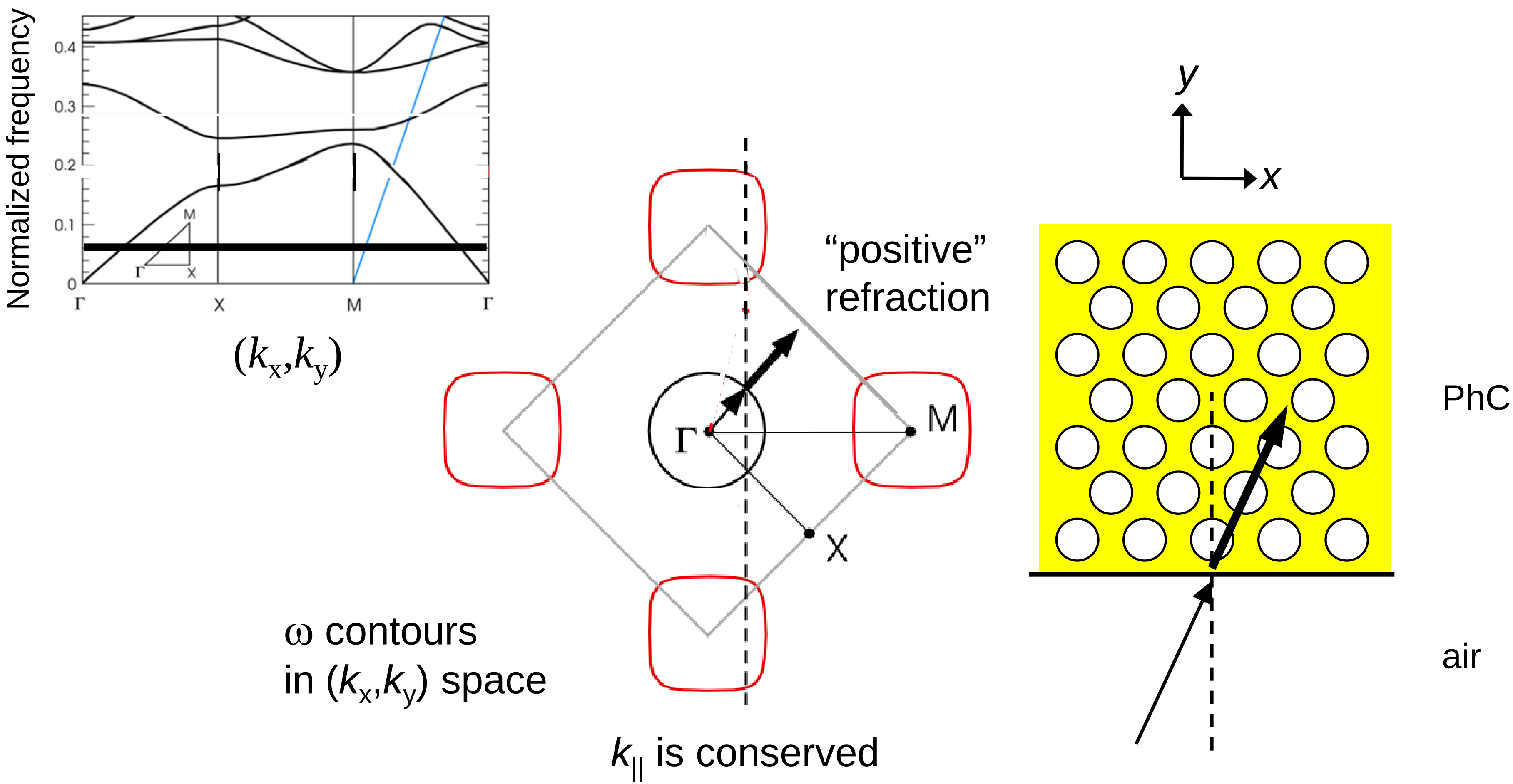
# Deviation from the static limit

Go to hell

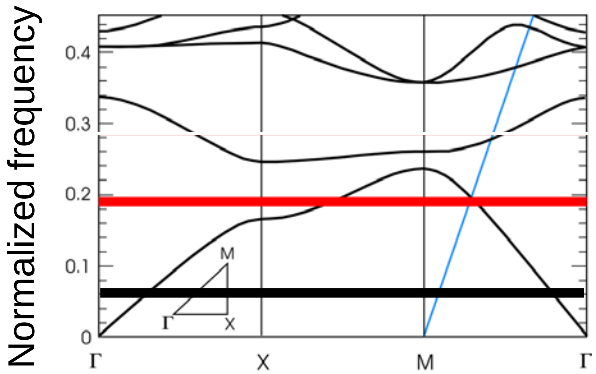
Bloch modes are no longer plane waves
Refraction/reflection laws at interfaces are no longer given by Fresnel coefficients
The dispersion relation is no longer ellipsoidal
Magneto-optic coupling
Artificial magnetism

$$< \mathbf{D} > = \varepsilon_{eff}(\omega, \mathbf{k}) < \mathbf{E} > + i\kappa_{eff}(\omega, \mathbf{k}) < \mathbf{H} >$$
$$< \mathbf{B} > = \mu_{eff}(\omega, \mathbf{k}) < \mathbf{H} > + i\kappa_{eff}(\omega, \mathbf{k}) < \mathbf{E} >$$

# Deviation from the static limit

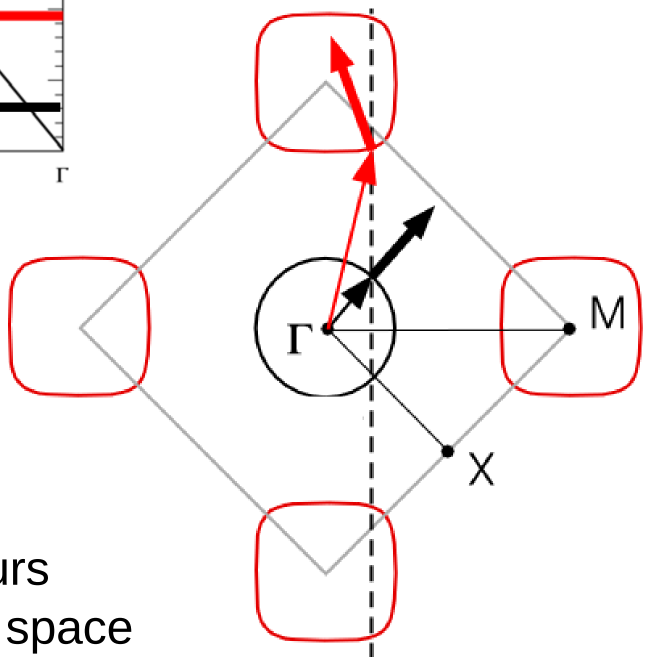


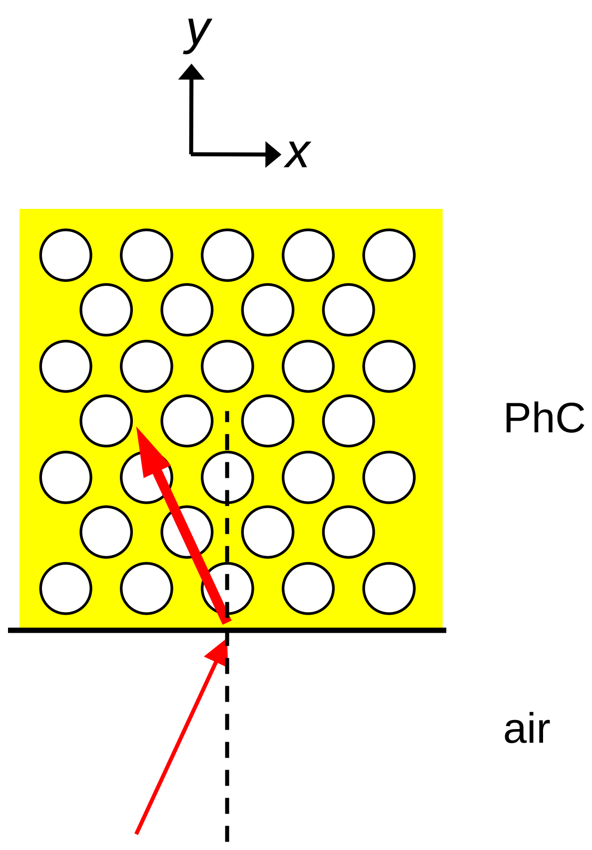
# Deviation from the static limit




$(k_x,k_y)$

"negative"
refraction

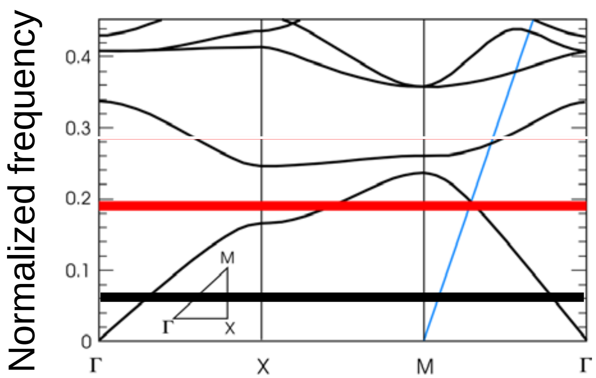


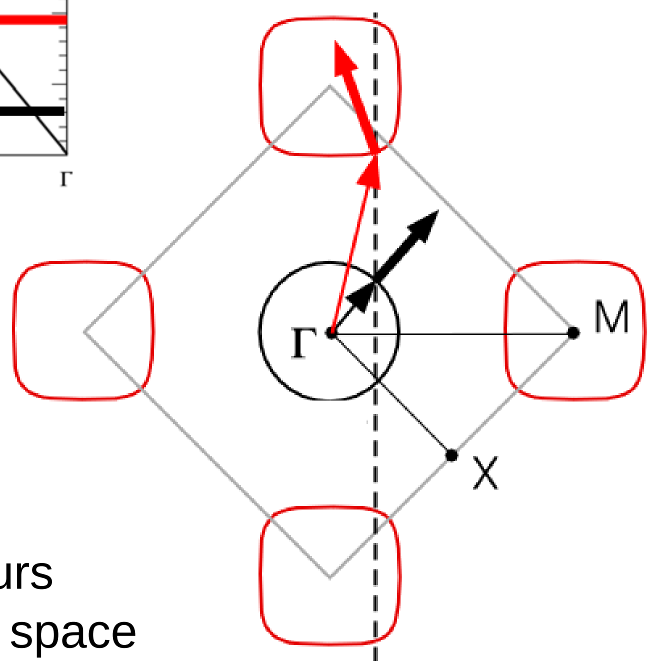

ω contours
in $(k_x,k_y)$ space

$k_{||}$ is conserved

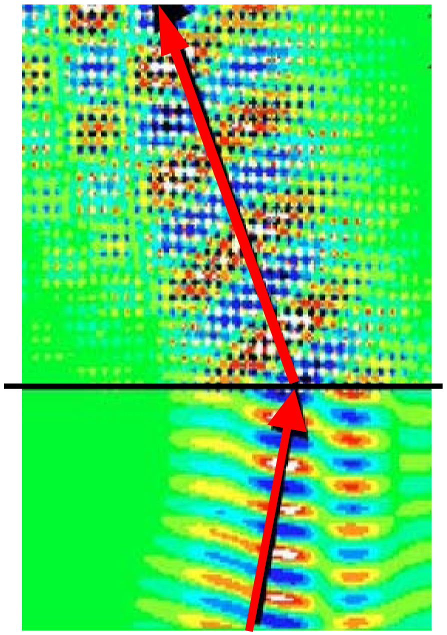
# Deviation from the static limit




$(k_x,k_y)$




Luo et al., PRB **68** (2003).



PhC

air

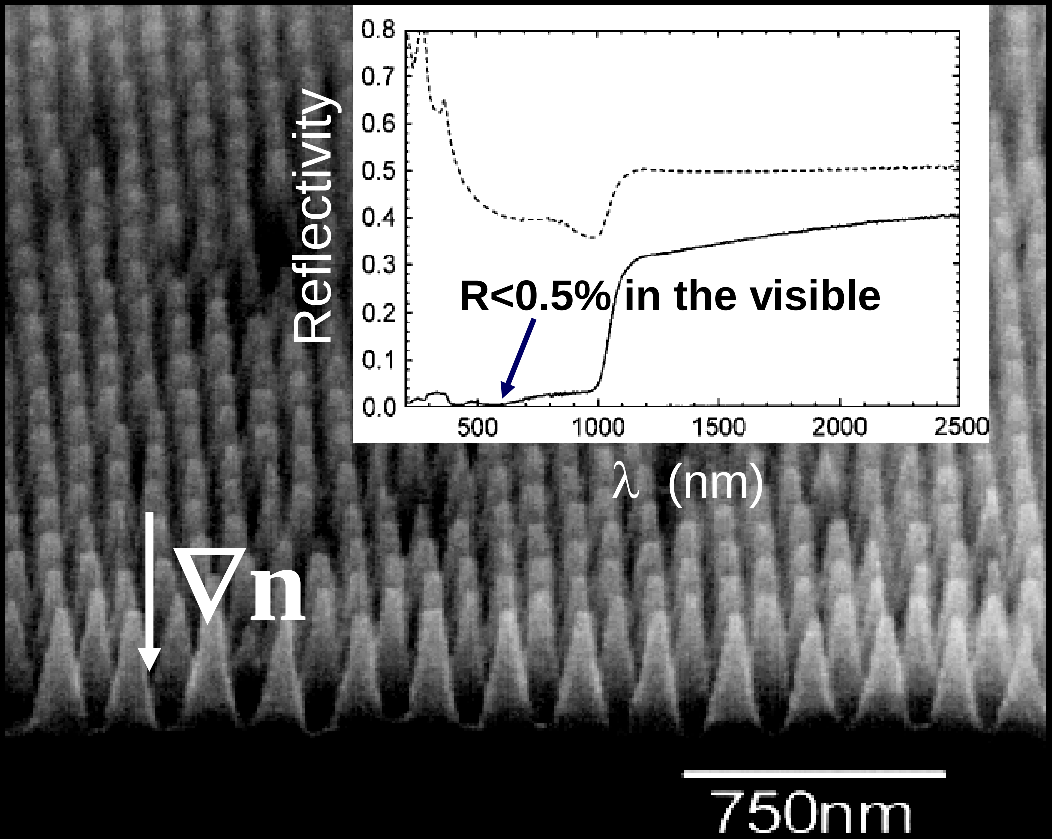
$\omega$ contours
in $(k_x,k_y)$ space

$k_{||}$ is conserved

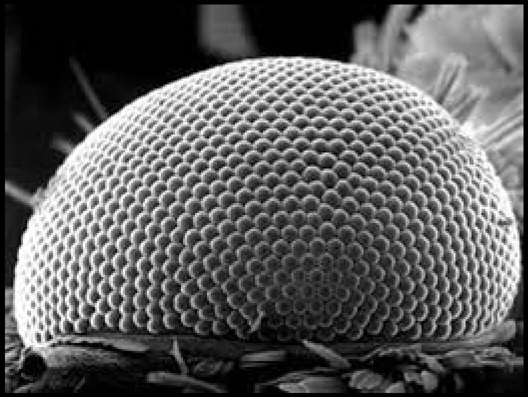
Here, using *positive* effective index but negative "effective mass"…

# Dielectric gradient metasurface

# Semiconductor anti-reflection

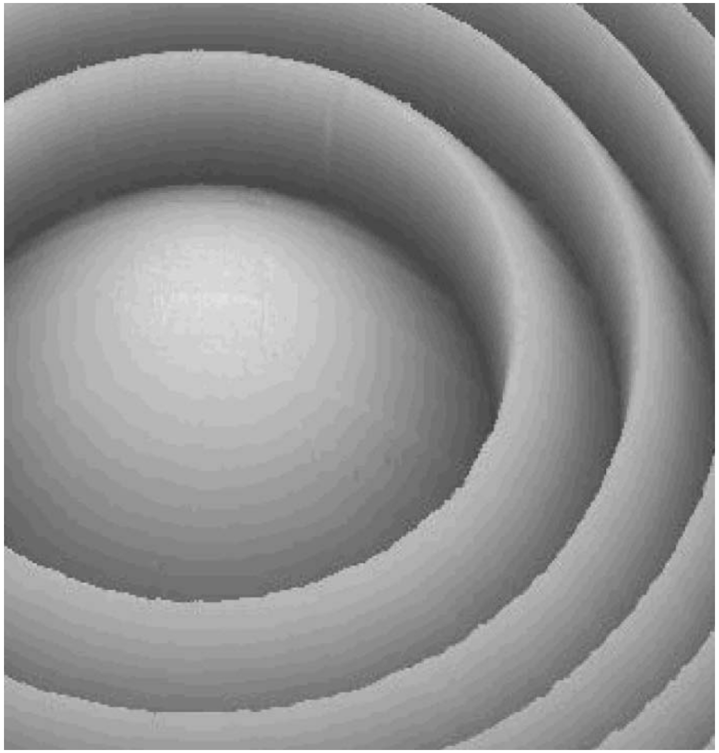



anechoic chamber

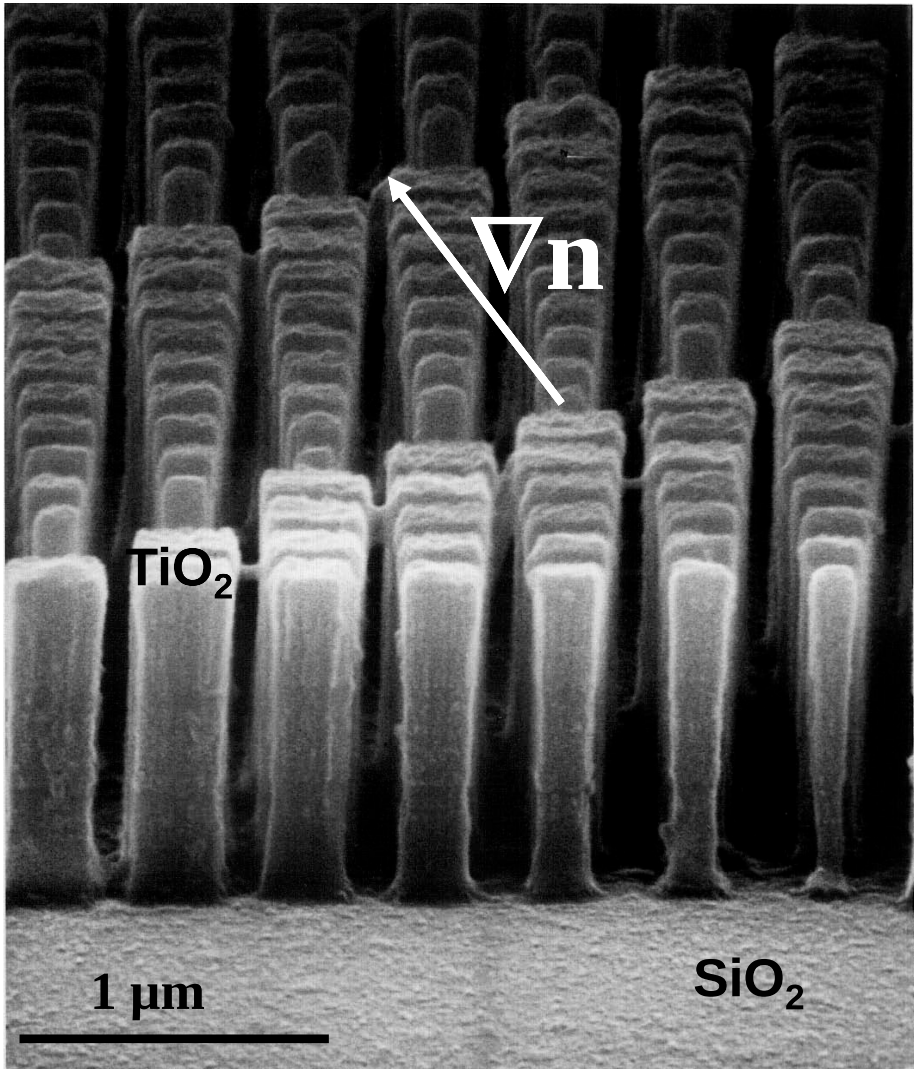


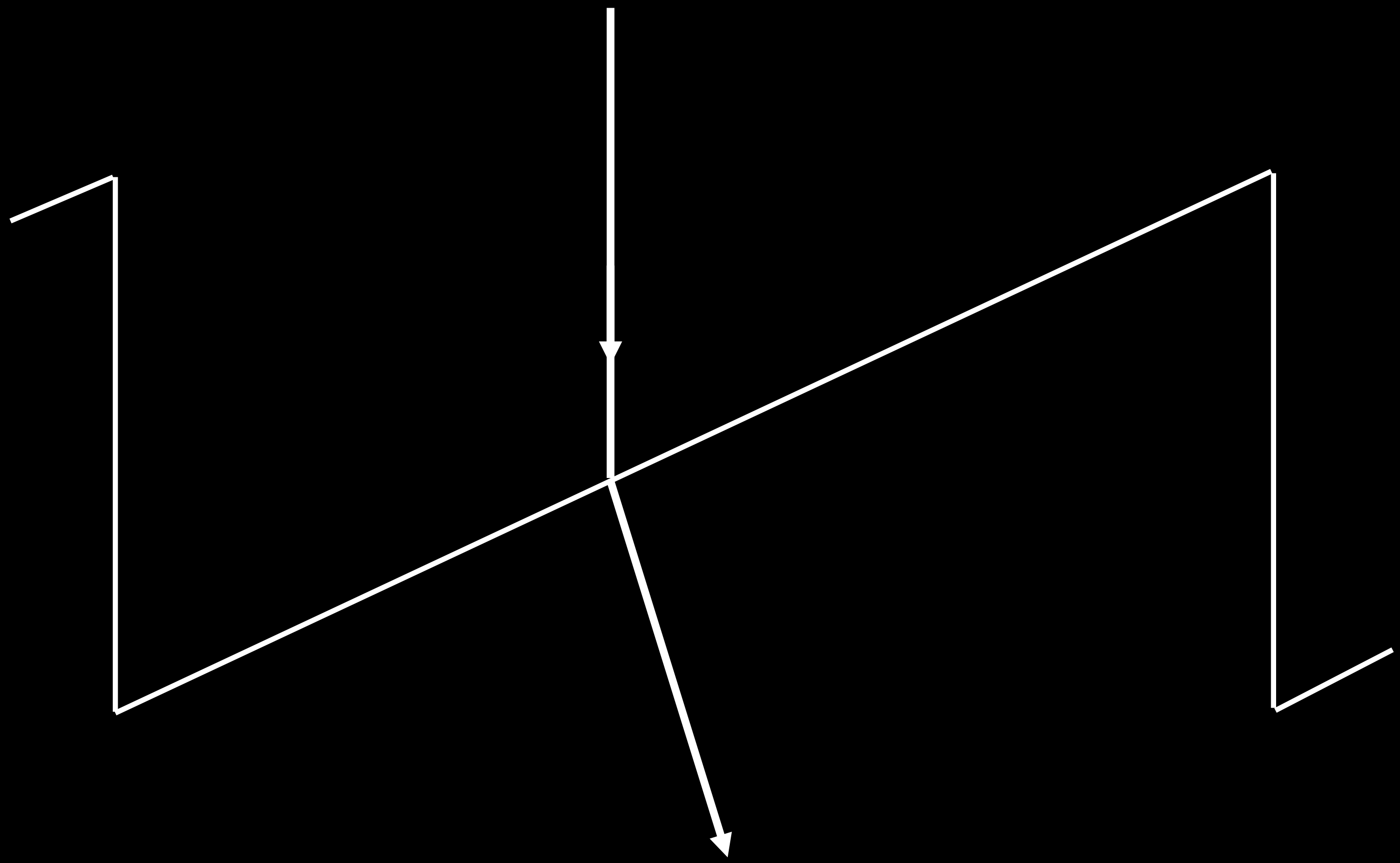
moth eye

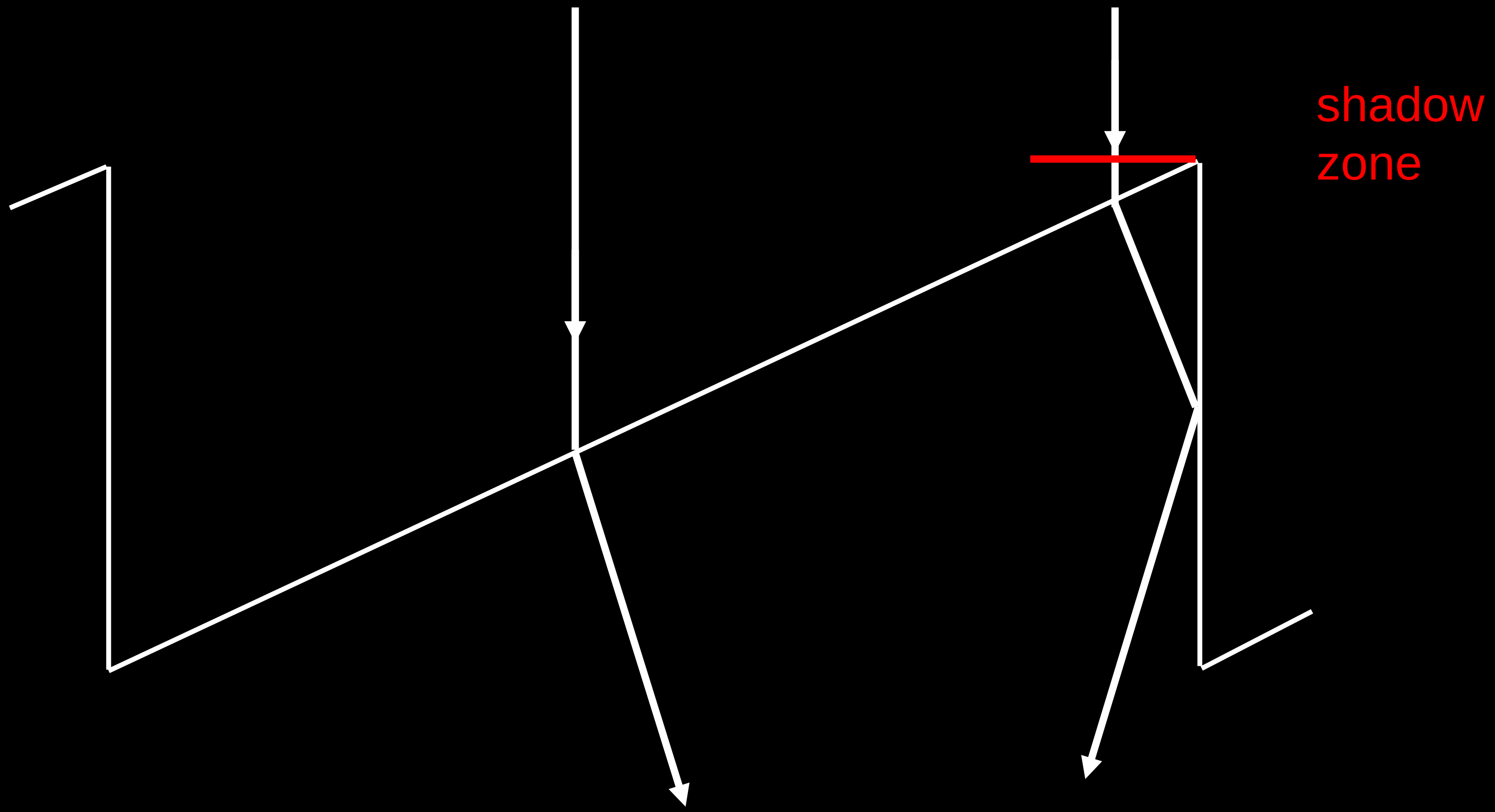
Y. Kanamori, M. Sasaki and K. Hane, Opt. Lett. **24** (2000).

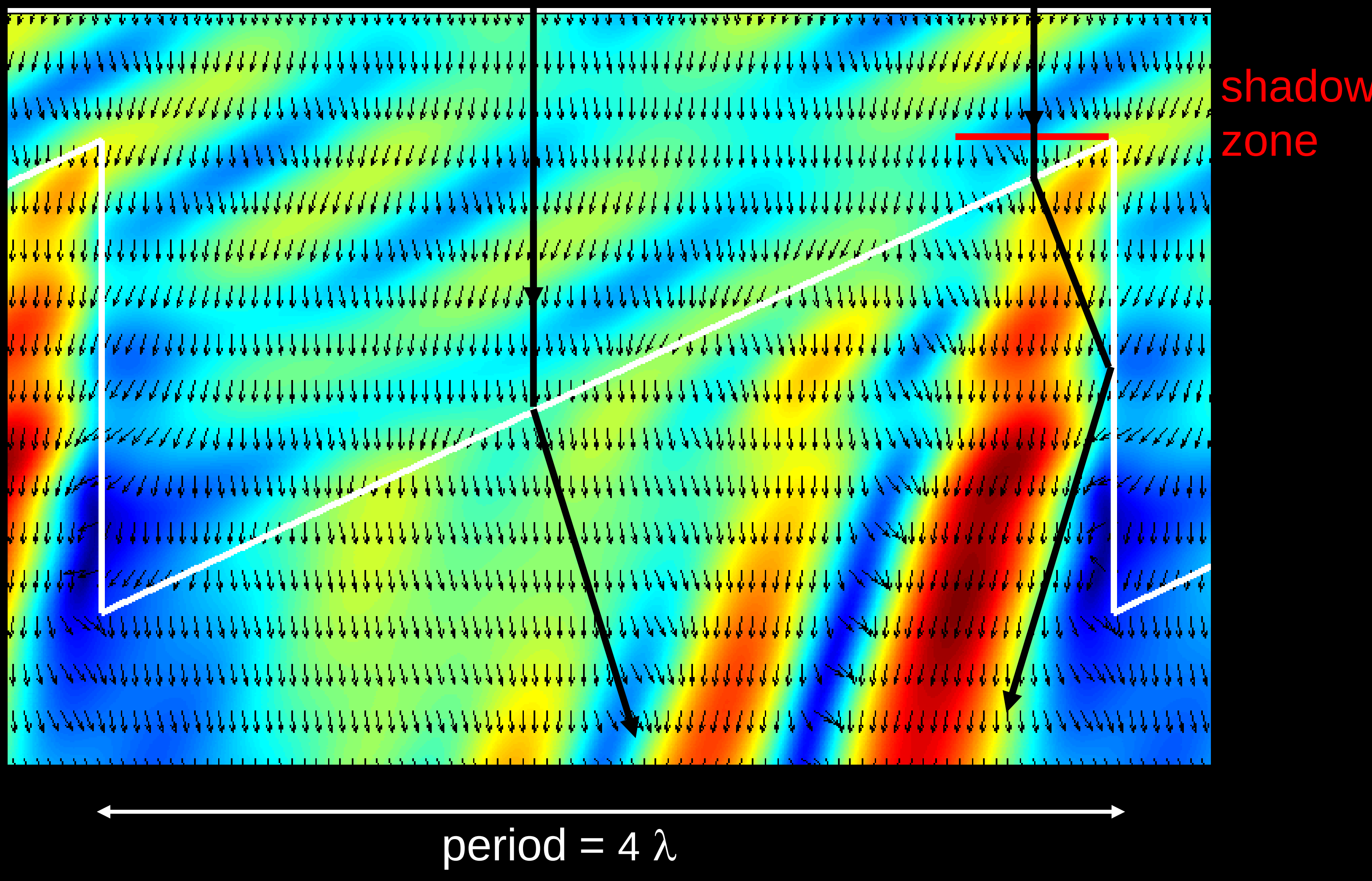


Fresnel lens

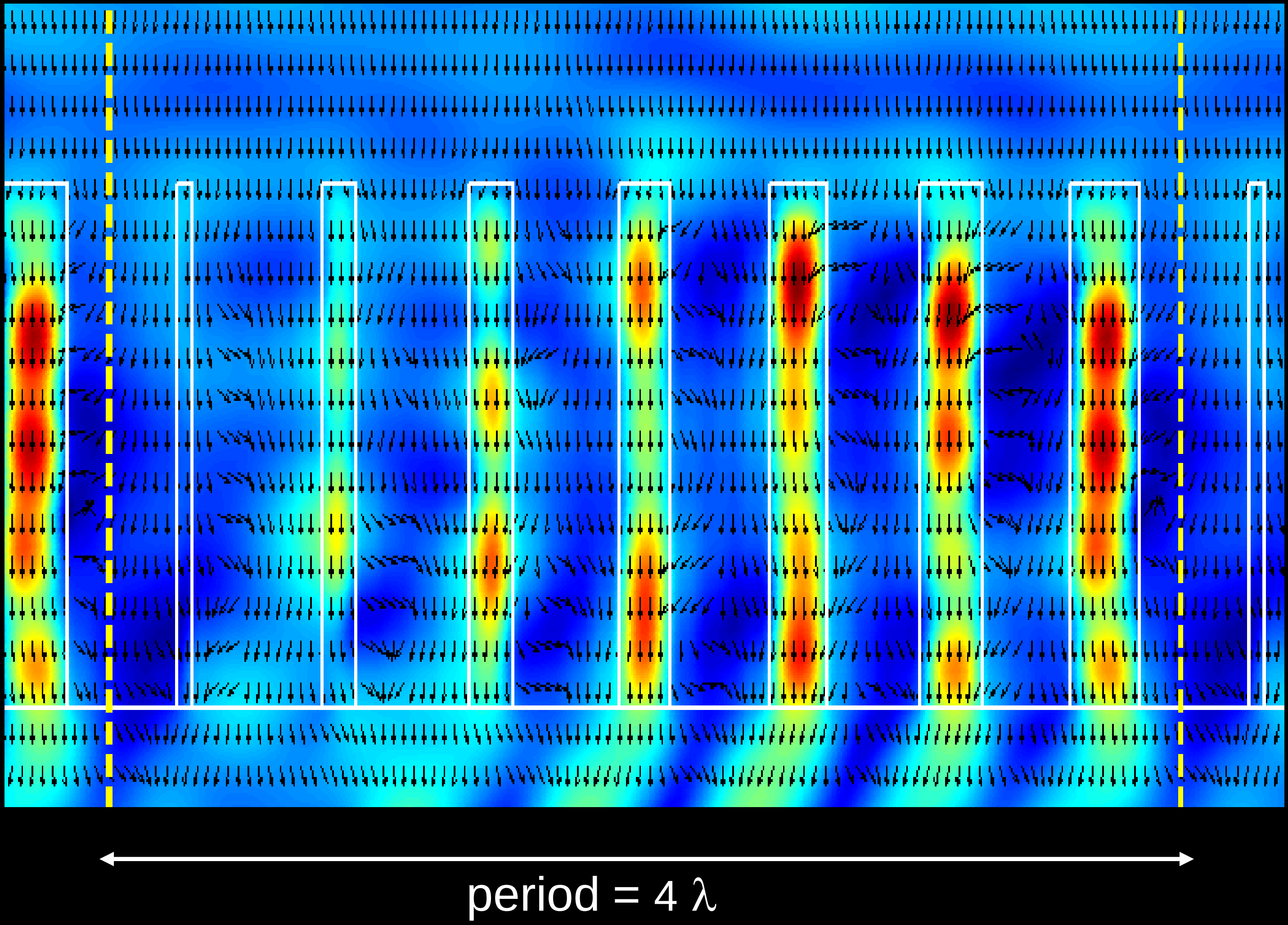




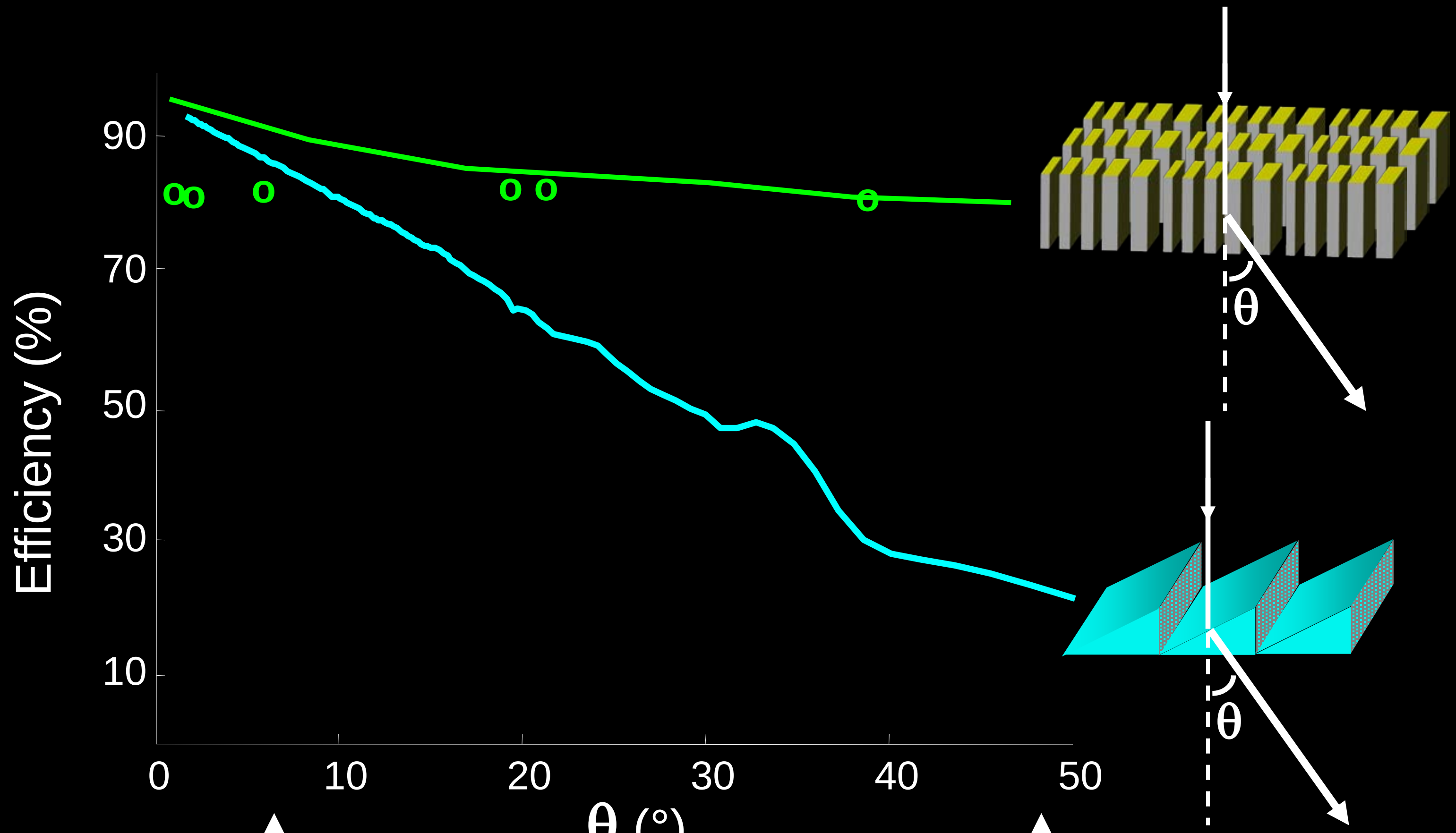

shadow
zone


shadow
zone
period = 4 λ

# Waveguiding effect



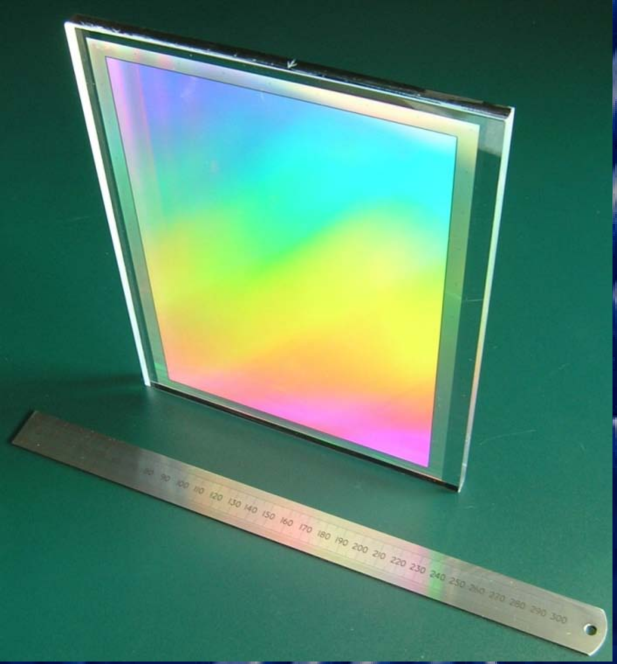
# Better efficiency than Echelette




large periods

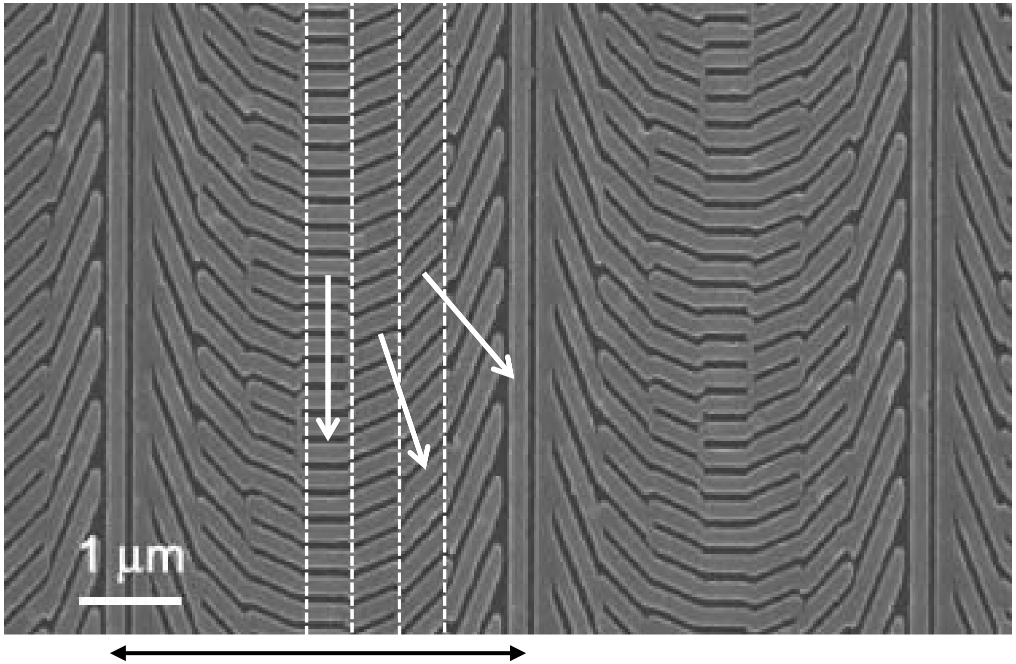
small periods

M.S. Lee et al., J. opt. 4, S119 (2002).

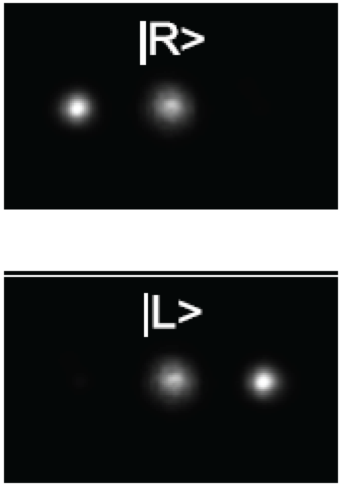



**Courtezy U.D. Zeitner, Fraunhofer Institut für Angewandte Optik und Feinmechanik, Jena.**
**Sent in space on 19. Dec. 2013 in the Gaia-satellite of the ESA**
**http://sci.esa.int/science-e/www/object/index.cfm?fobjectid=44093**

Half wave plate: $k_0(n_e-n_o)\,t = \pi$




period = 3 λ

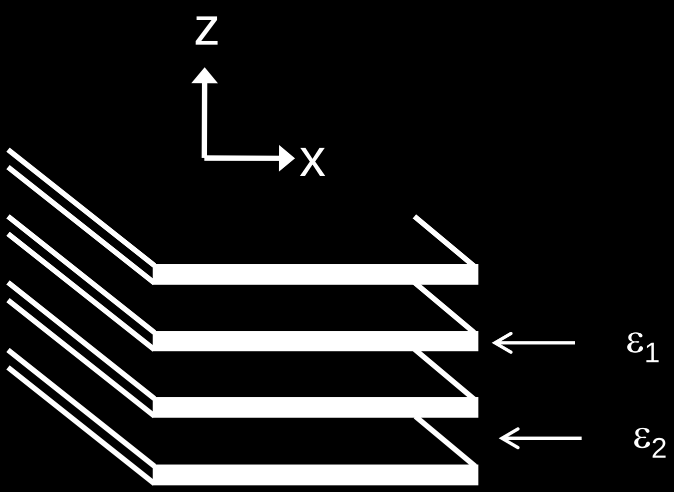



D. Lin et al., Science 345, 6194 (2014).

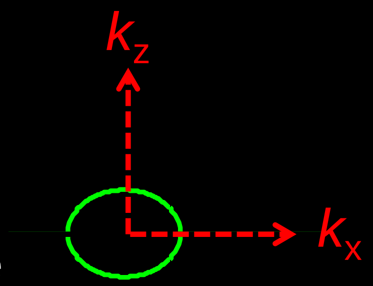
# Metallic metamaterials

# Hyperbolic media

$$\frac{(k_z)^2}{\langle\varepsilon\rangle} + \frac{(k_x)^2}{\langle 1/\varepsilon\rangle^{-1}} = (\omega/c)^2$$




ω contours
in ($k_x$,$k_y$) space



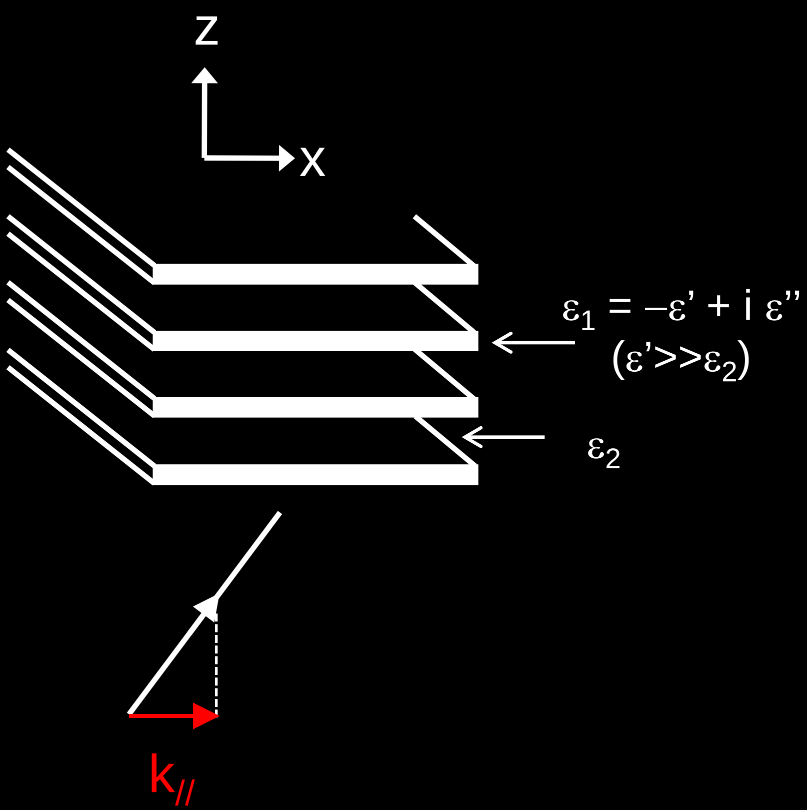

ellipsoid for
dielectric stack

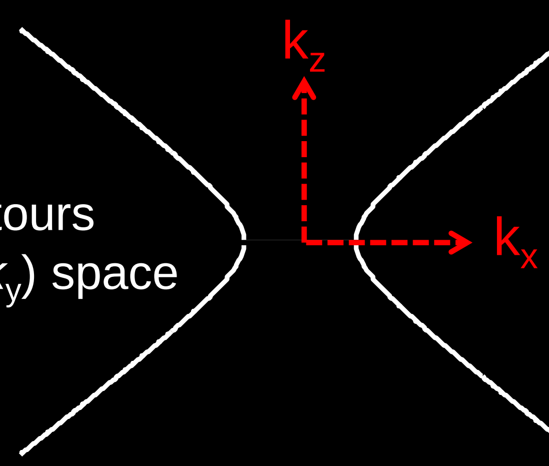
# Hyperbolic media

$$\frac{(k_z)^2}{\langle\varepsilon\rangle} + \frac{(k_x)^2}{\langle 1/\varepsilon\rangle^{-1}} = (\omega/c)^2$$

$\langle\varepsilon\rangle \approx f\varepsilon_1 < 0$ (metal-like)

$\langle 1/\varepsilon\rangle^{-1} \approx \varepsilon_2/(1-f) > 0$ (dielectric-like)



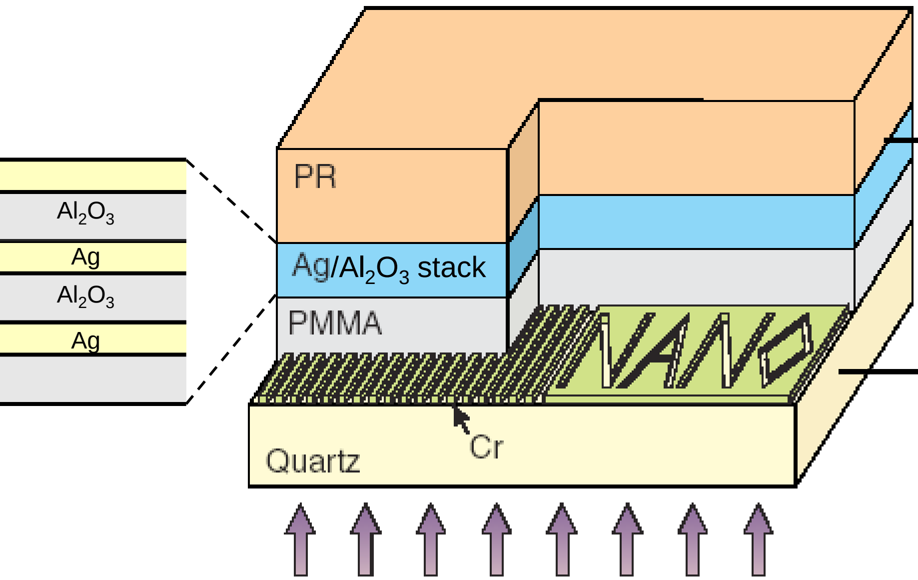

$\omega$ contours in $(k_x,k_y)$ space

metal/dielectric stack

# Hyperlensing with hyperbolic media

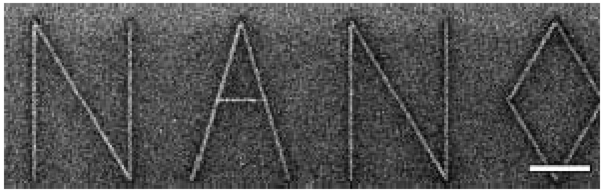
Experiment

Result




365 nm illumination

Image achieved after PR development



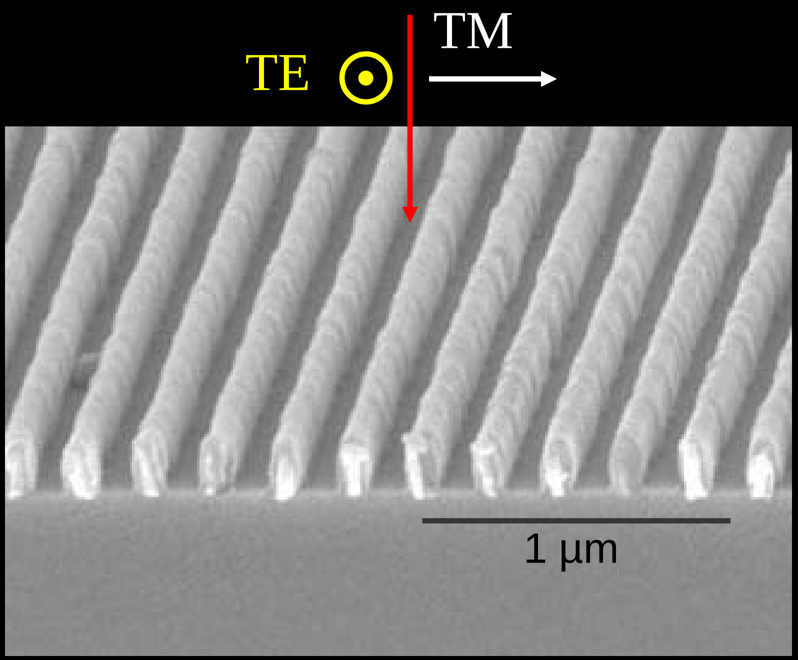
SEM image of the Cr mask

90 nm (λ/4) resolution claimed

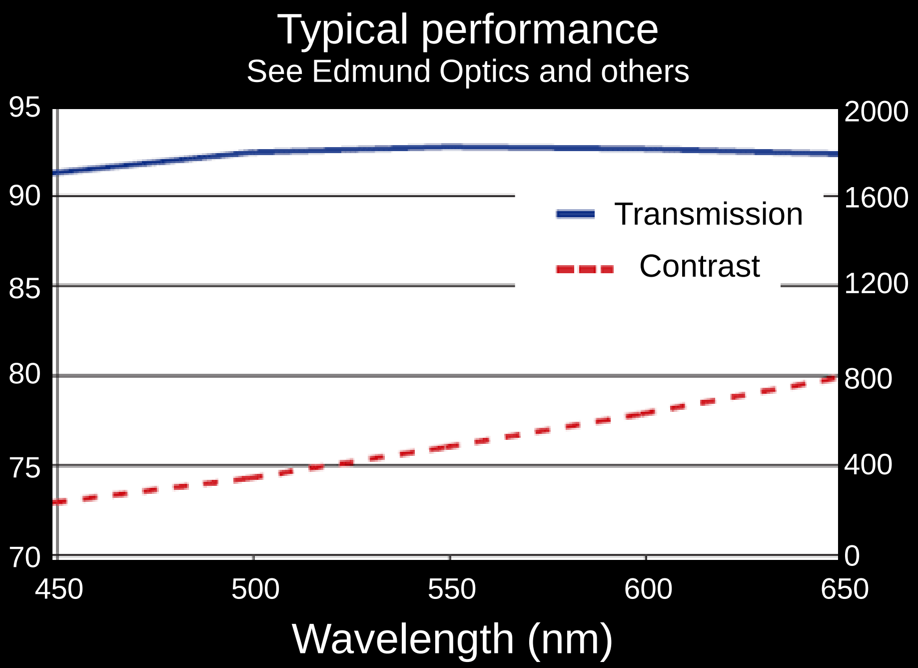
Xiang Zhang's group Berkeley, Science **308**, 534 (2005) & **315**, 1686 (2007)

# Wire-grid at optical frequencies

TE (ordinary wave):
$(n_o)^2 = \langle\varepsilon\rangle$ (metal-like)
TM (extraordinary wave):
$(n_e)^2 = \langle 1/\varepsilon\rangle^{-1}$ (dielectric-like)

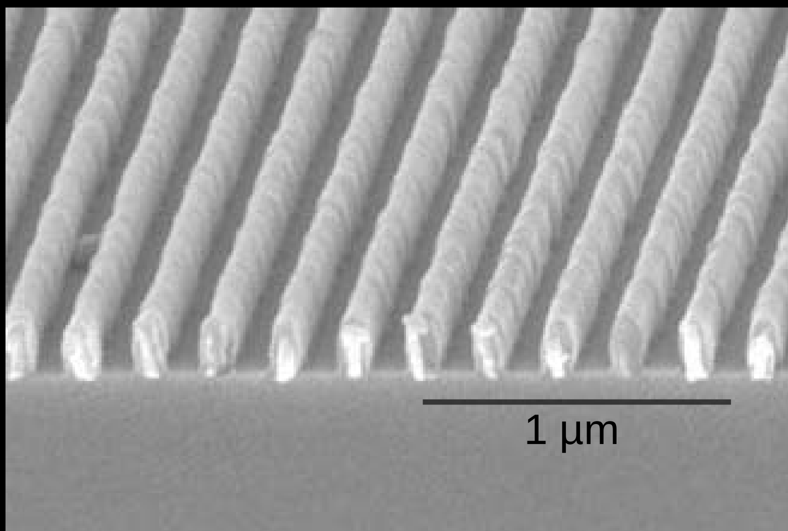


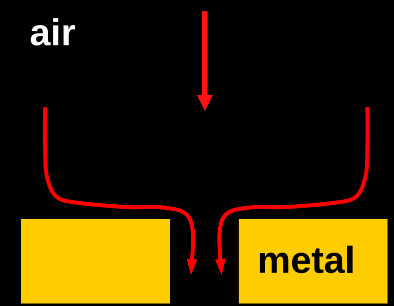

H. Tamada et al., Opt. Lett. 22, 419 (1997)

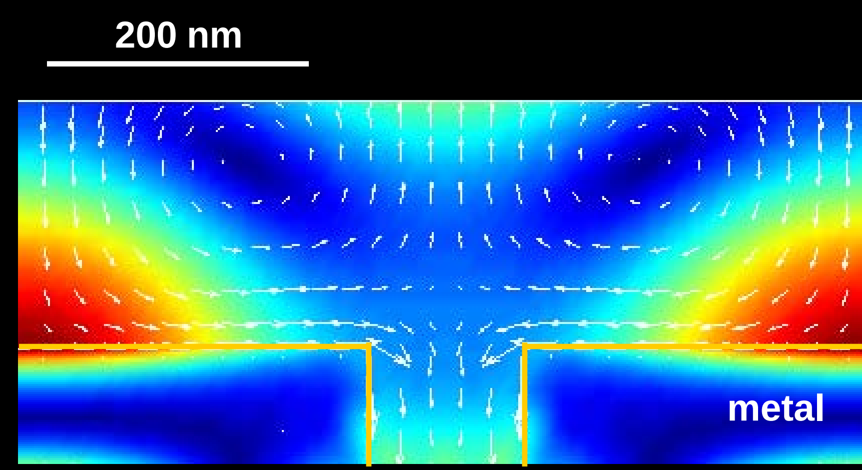



Principle used by Hertz for analysing the newly discovered radio waves

# Optical wire-grid polarizer




The physics of the polarization effect is more related to the extraordinary optical transmission than to an averaging process involving a metamaterial, like in Hertz experiment.

# Artificial media with ε & μ<0

Electromagnetism of media with ε and μ<0

1. is safe (no violation of basic principles)
2. offers new exciting perspectives
3. may be investigated in man-made materials

# Electrodynamics in media with ε & μ<0

$\nabla\times\mathbf{E} = -i\omega\,\mu_0\mu\,\mathbf{H}$

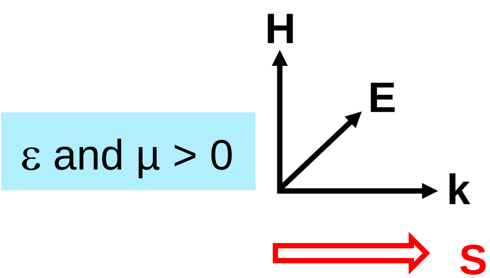
$\nabla\times\mathbf{H} = \;\; i\omega\,\varepsilon_0\varepsilon\,\mathbf{E}$

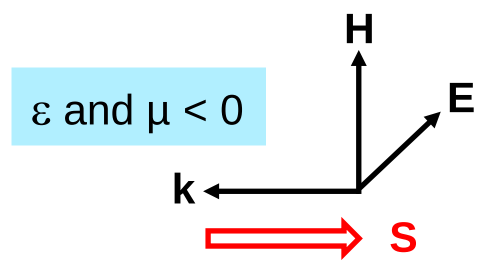

Mathematics in Maxwell's equations are unchanged:
the electromagnetic modes are plane waves again: $\exp(i\omega t - i\mathbf{kr})$

$\mathbf{k}\times\mathbf{H} = \omega\varepsilon_0\varepsilon\,\mathbf{E}$ and $\mathbf{k}\times\mathbf{E} = -\omega\mu_0\mu\,\mathbf{H}$

Poynting vector unchanged: $\mathbf{S} = \mathbf{E}\times\mathbf{H}$

Impedance unchanged : sqrt(εμ)

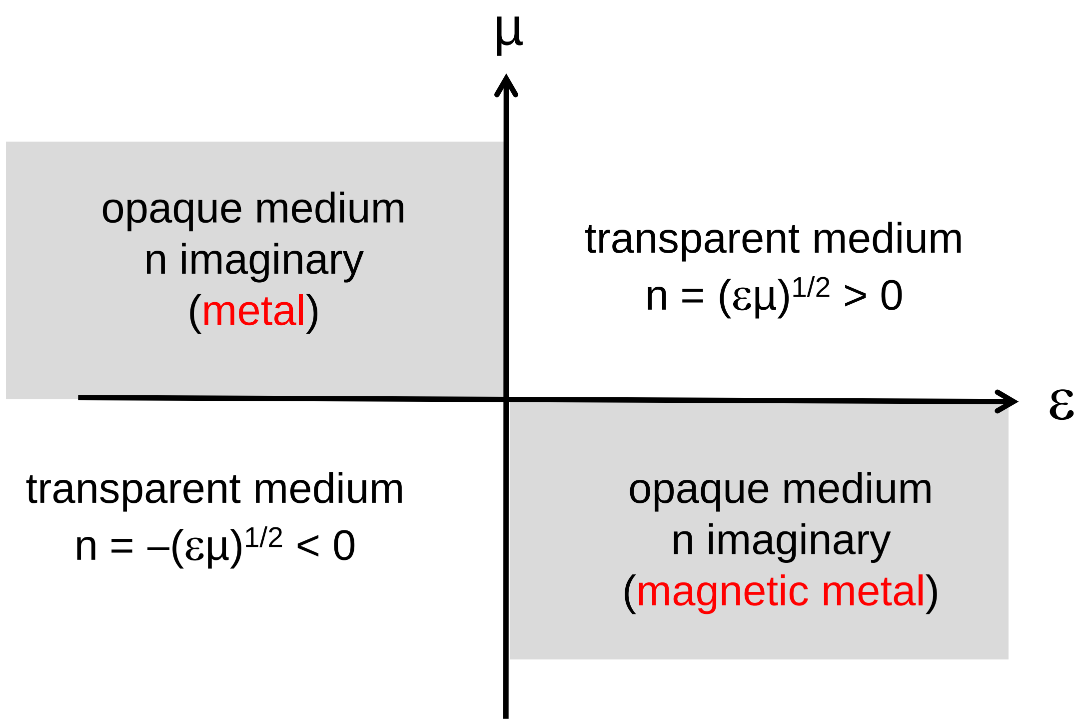



Just change n in –n :

- $n = (\varepsilon\mu)^{1/2}$ if $\varepsilon,\mu > 0$
- $n = -(\varepsilon\mu)^{1/2}$ if $\varepsilon,\mu < 0$

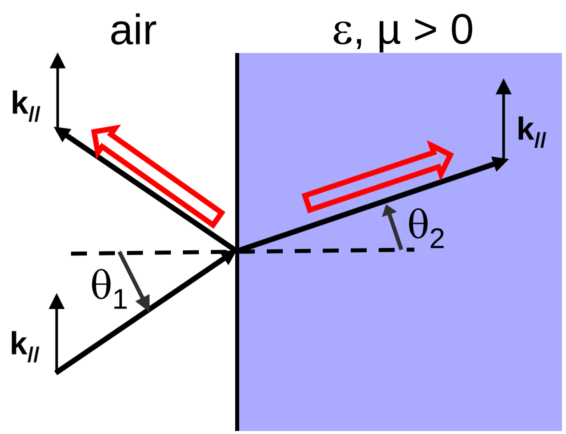
# Negative index



# Negative refraction

•Fields continuities on the interface : $\exp(i\omega_1 t - i\mathbf{k}_1\mathbf{r}) = \exp(i\omega_2 t - i\mathbf{k}_2\mathbf{r})$, for $z = 0$.

•Outgoing wave conditions : energy flows outwards



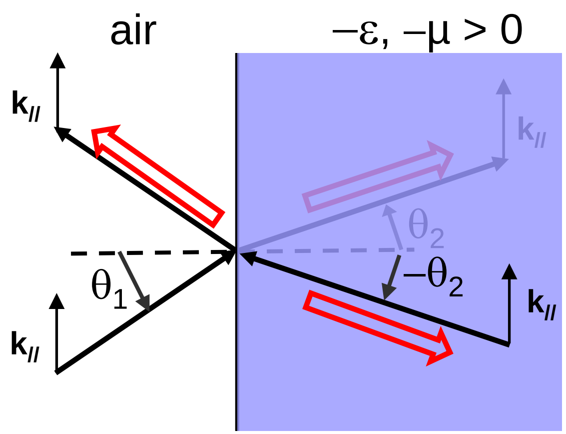

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

<u>Snell law still applies</u>

# Negative refraction

•Fields continuities on the interface : $\exp(i\omega_1 t - i\mathbf{k}_1\mathbf{r}) = \exp(i\omega_2 t - i\mathbf{k}_2\mathbf{r})$, for $z = 0$.

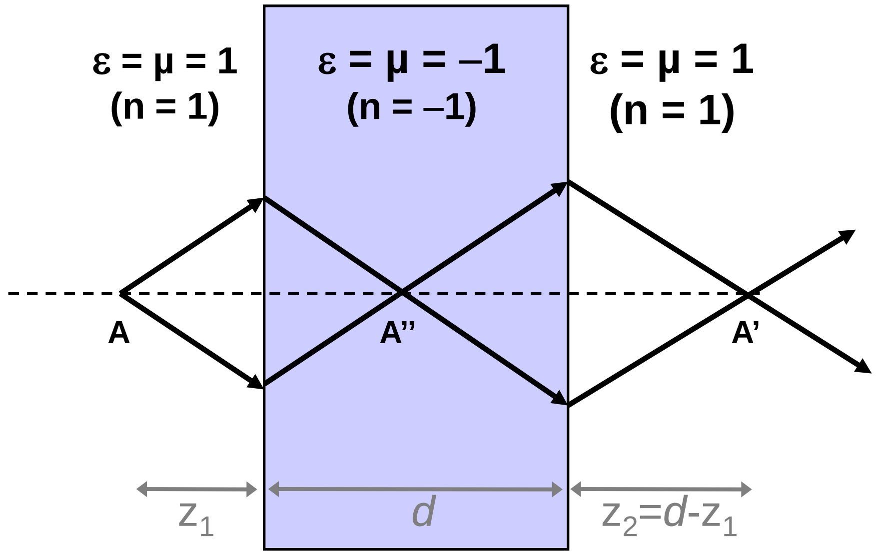
•Outgoing wave conditions : energy flows outwards




$$n_1\sin(\theta_1) = -n_2\sin(-\theta_2)$$

<u>Snell law still applies</u>

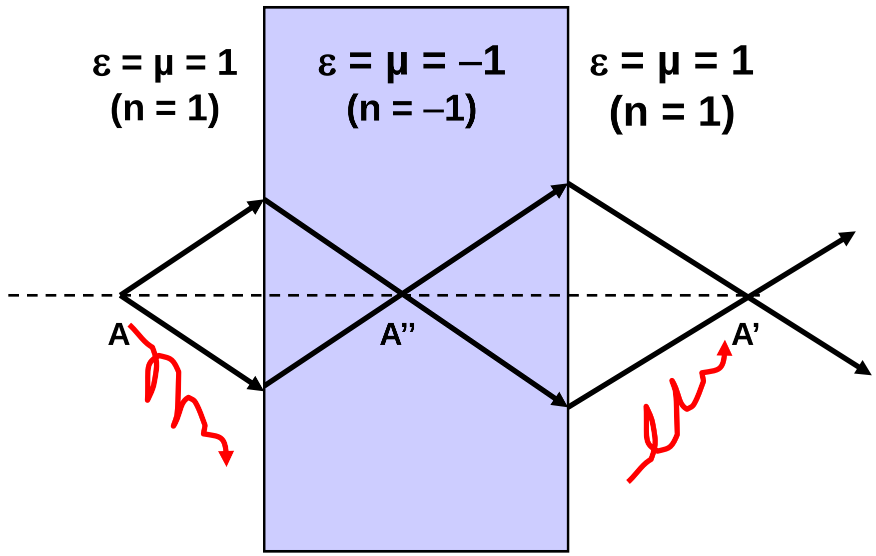
# Veselago's flat lens




•the optical path from the external focus to the internal focus is zero; it is extremal like in classical lens design,
•the lens has no optical axis,
•the magnification is always 1,
•the geometrical aberrations are null; the point to point correspondence is perfect,
•All light goes through, no back-reflection since the impedance of the medium is a perfect match to free space $Z = Z_0(\mu/\varepsilon)^{1/2}$ ($Z_0 = (\mu_0/\varepsilon_0)^{1/2}$ being the impedance of vacuum).

V.G. Veselago, Soviet. Phys. Usp **10**, 509 (1968).

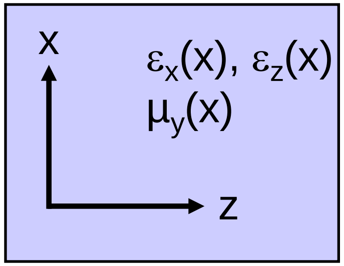
# Exciting perspective: the perfect lens




"With a conventional lens sharpness of the image is always limited by the wavelength of light. An unconventional alternative to a lens, a slab of negative refractive index material, has the power to focus all Fourier components of a 2D image, even those that do not propagate in a radiative manner."

J. Pendry, *The perfect lens*, Phys. Rev. Lett. 85, 3966 (2000).

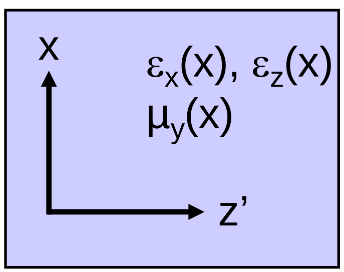
# Transformation optics : 2D case




- $\partial_z E_x - \partial_x E_z = -i\omega\, \mu_y\, H_y$
- $-\partial_z H_y = i\omega\, \varepsilon_x\, E_x$
- $\partial_x H_y = i\omega\, \varepsilon_z\, E_z$

$z' = \alpha z$

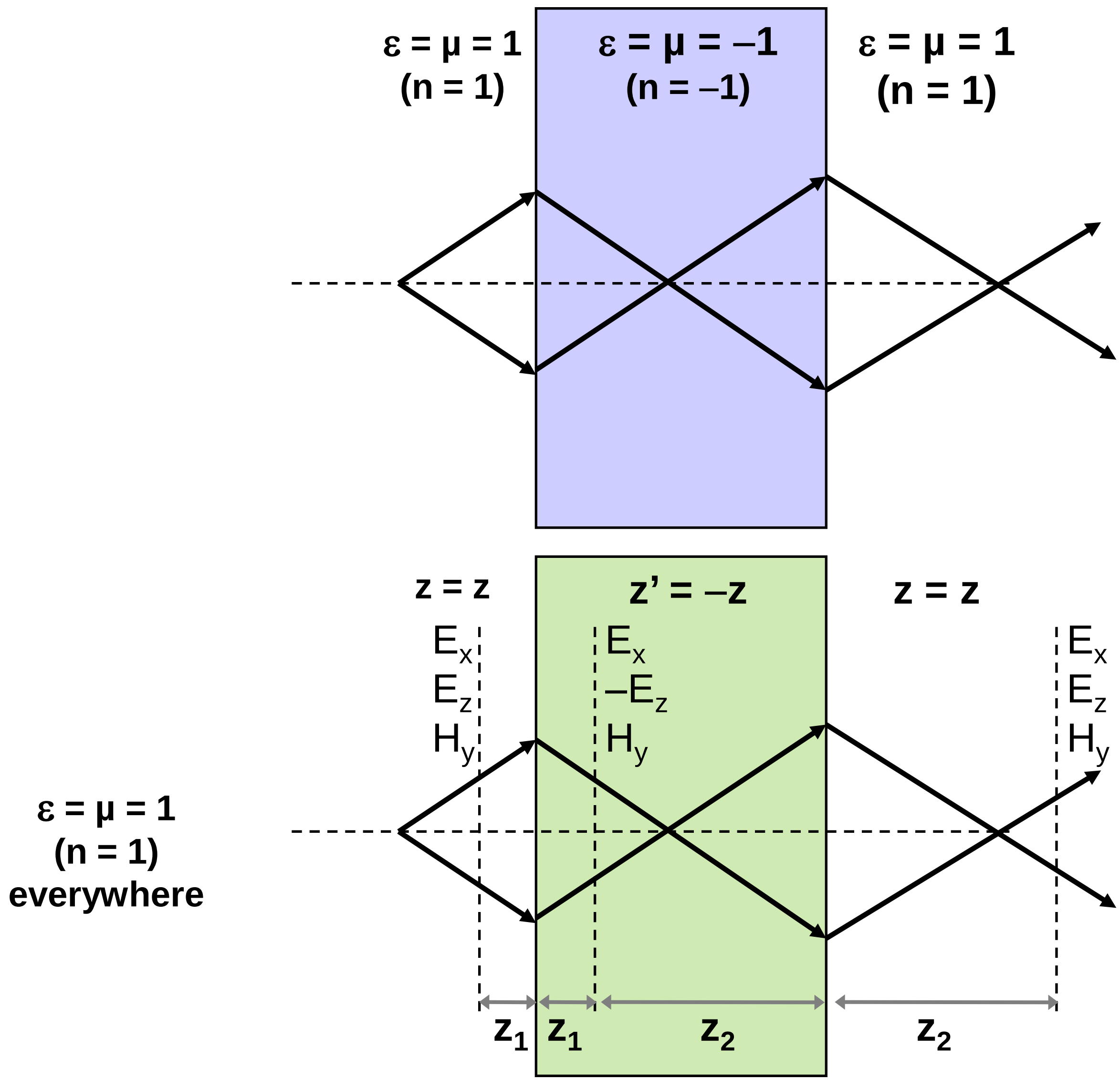
- $\alpha \partial_{z'} E_x - \partial_x E_z = -i\omega \mu_y\, H_y$
- $-\alpha \partial_{z'} H_y = i\omega\, \varepsilon_x\, E_x$
- $\partial_x H_y = i\omega\, \varepsilon_z\, E_z$



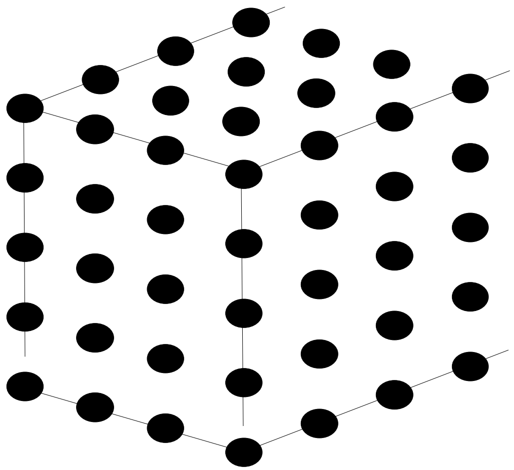

- $\partial_{z'} E_x - \partial_x (E_z/\alpha) = -i\omega\, (\mu_y/\alpha)\, H_y$
- $-\partial_{z'} H_y = i\omega\, (\varepsilon_x/\alpha)\, E_x$
- $\partial_x H_y = i\omega\, (\alpha\varepsilon_z)\, (E_z/\alpha)$

If $(E_x\ E_z\ H_y)$ is a solution of Maxwell's equations in a medium with $\varepsilon_x(x)$, $\varepsilon_z(x)$, $\mu_y(x)$,
Then
$(E_x\ E_z/\alpha\ H_y)$ is a solution of Maxwell's equations in a medium with $\varepsilon_x(x)/\alpha$, $\alpha\varepsilon_z(x)$, $\mu_y(x)/\alpha$.

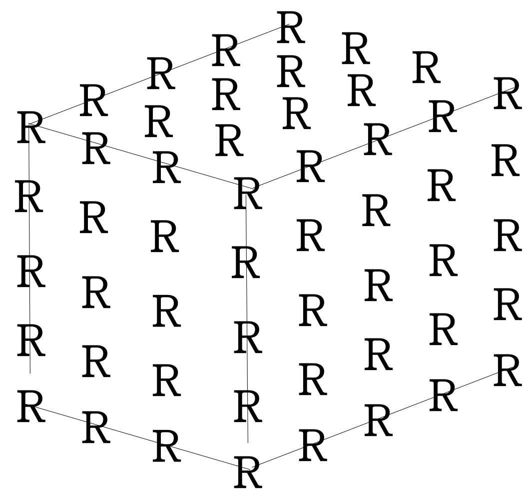
# The perfect lens



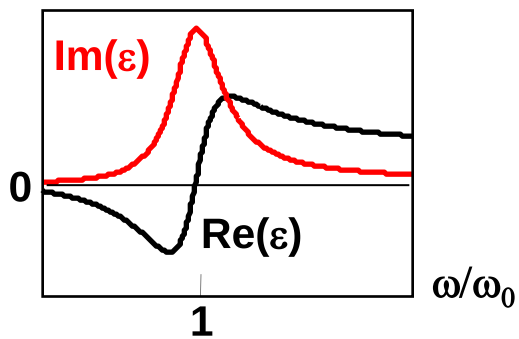
# How to make a media with ε & μ <0?

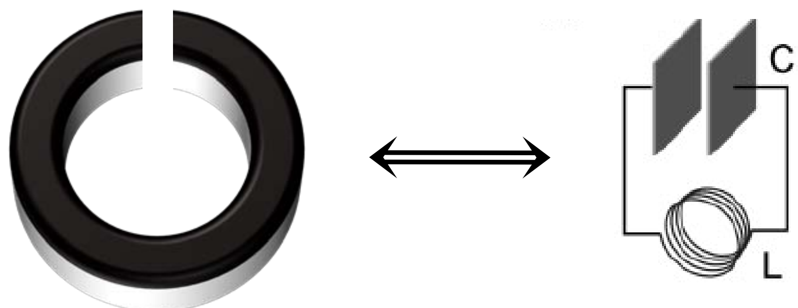


Real atoms with dielectric resonance
create negative electric polarisability



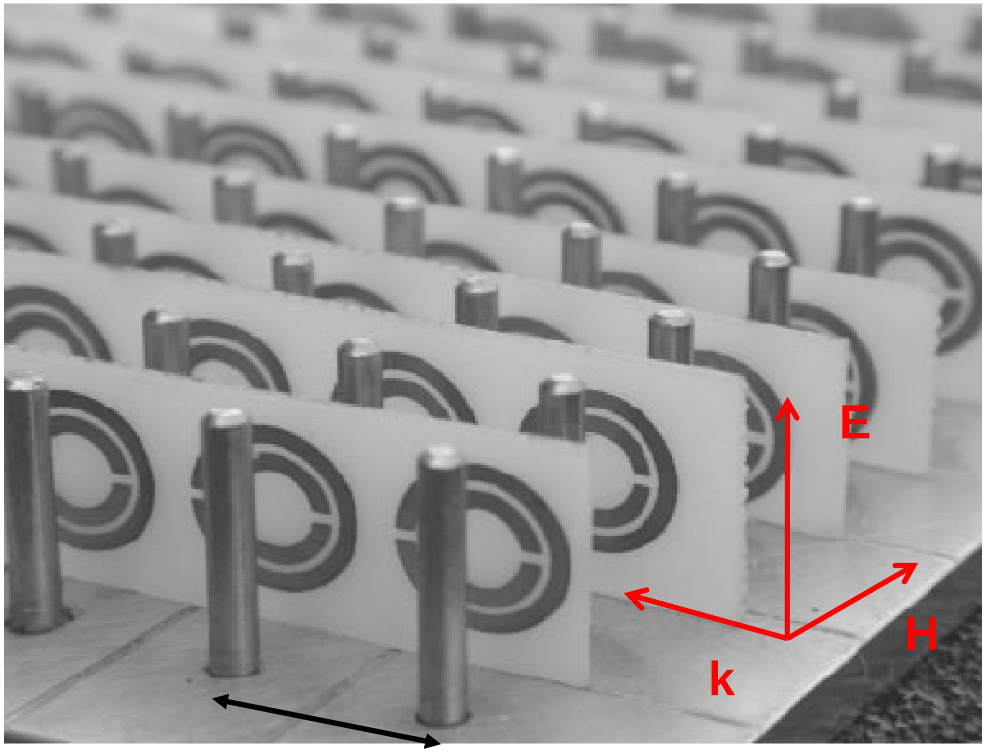
Metaatoms with a magnetic resonance



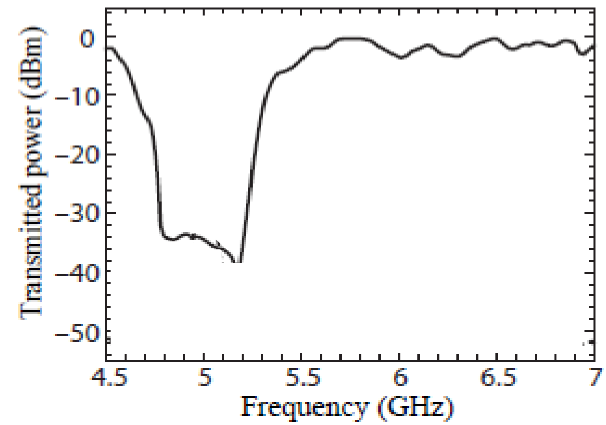

$\omega_0 = (LC)^{-1/2}$

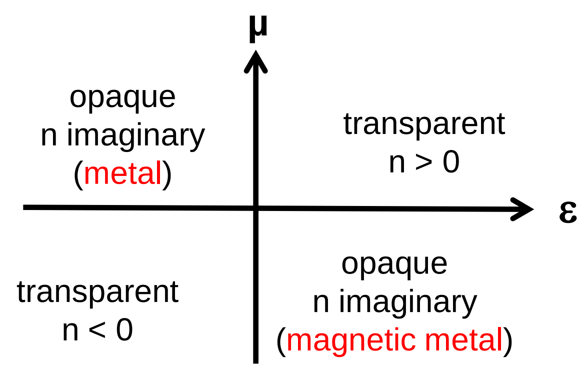
# Left-handed materials at λ = 60 mm




*a* = 8 mm
(λ = 60 mm)

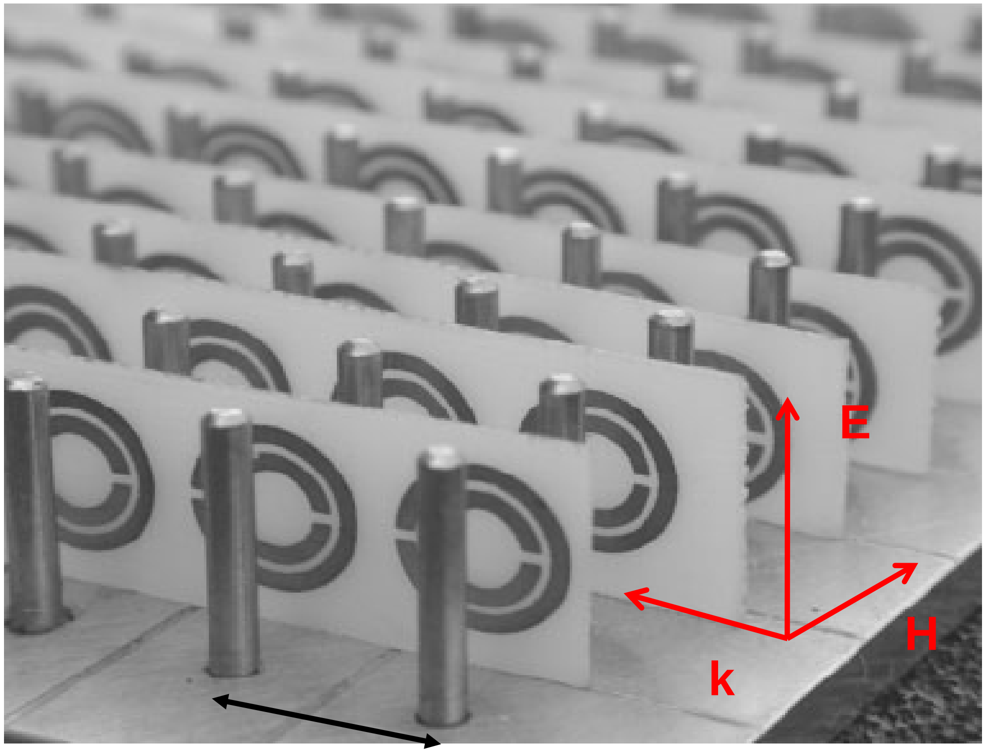
**First measurement: SRR only**



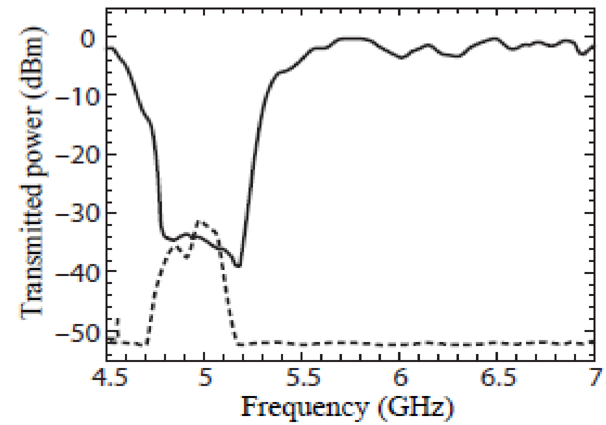

D. Smith et al., "Composite medium with simultaneously negative permeability and permittivity", PRL **84**, 4184 (2000).

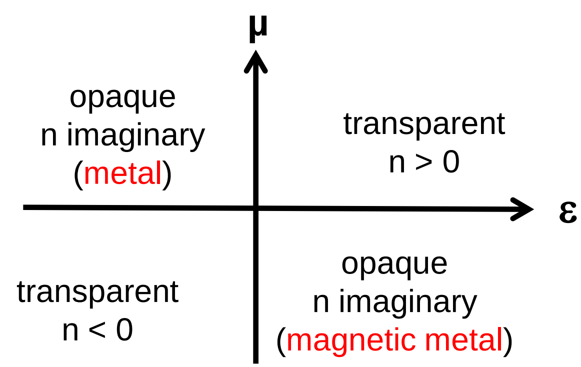
# Left-handed materials at λ = 60 mm




$a$ = 8 mm
(λ = 60 mm)

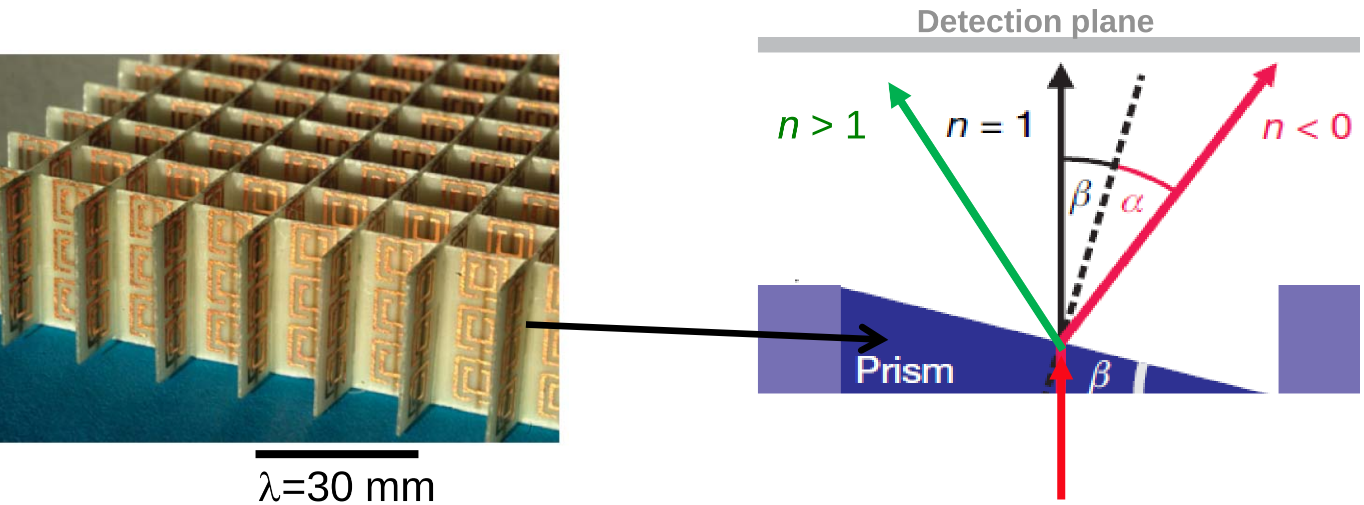
**Second measurement: SRR +rods**

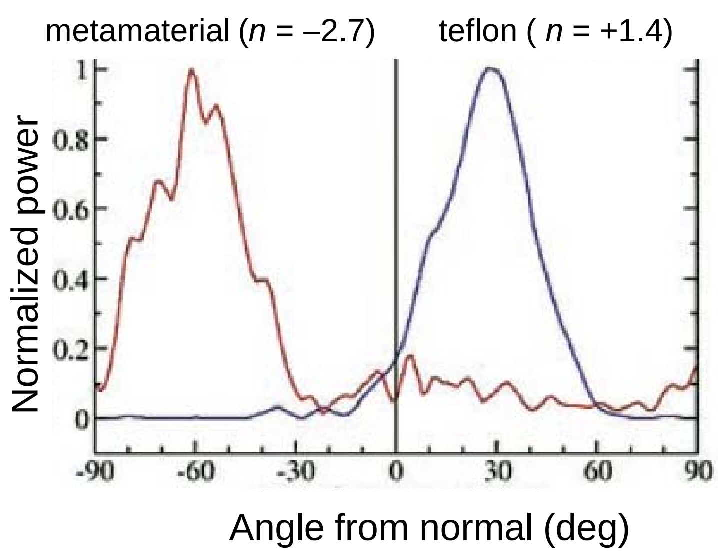



D. Smith et al., "Composite medium with simultaneously negative permeability and permittivity", PRL **84**, 4184 (2000).

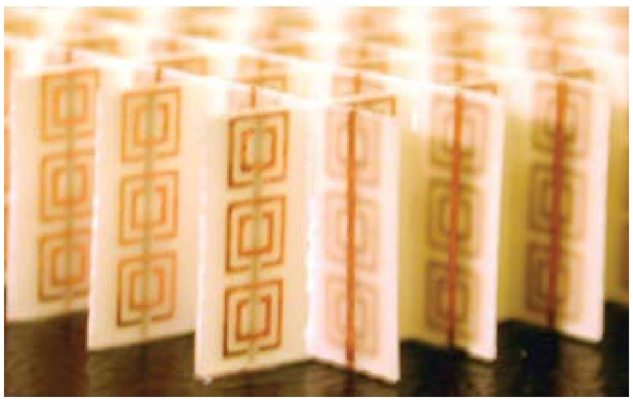
# Negative refraction



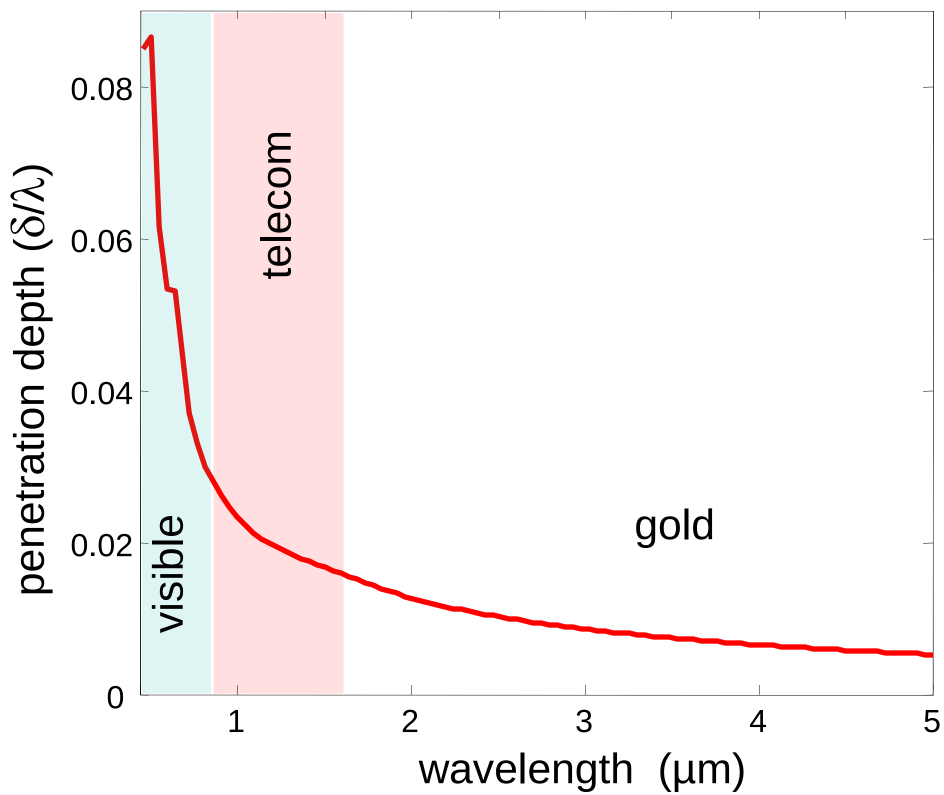

R. A. Shelby et al., "Experimental verification of a negative index of refraction", Science **292**, 77 (2001).

# Optical left-handed materials

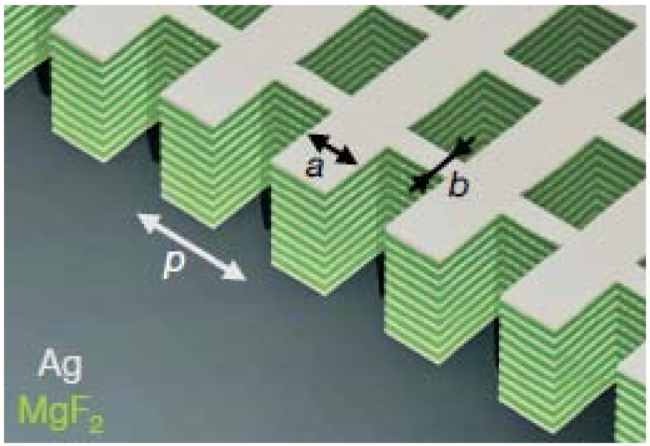


RLC equivalent-Circuit-Model

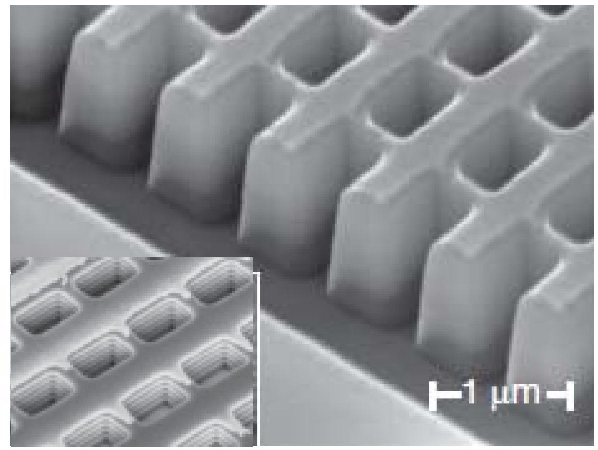
- no analytical model : only intuitive arguments
- periods ~ $\lambda/2$
- serious loss and manufacturing problems



# Optical left-handed materials

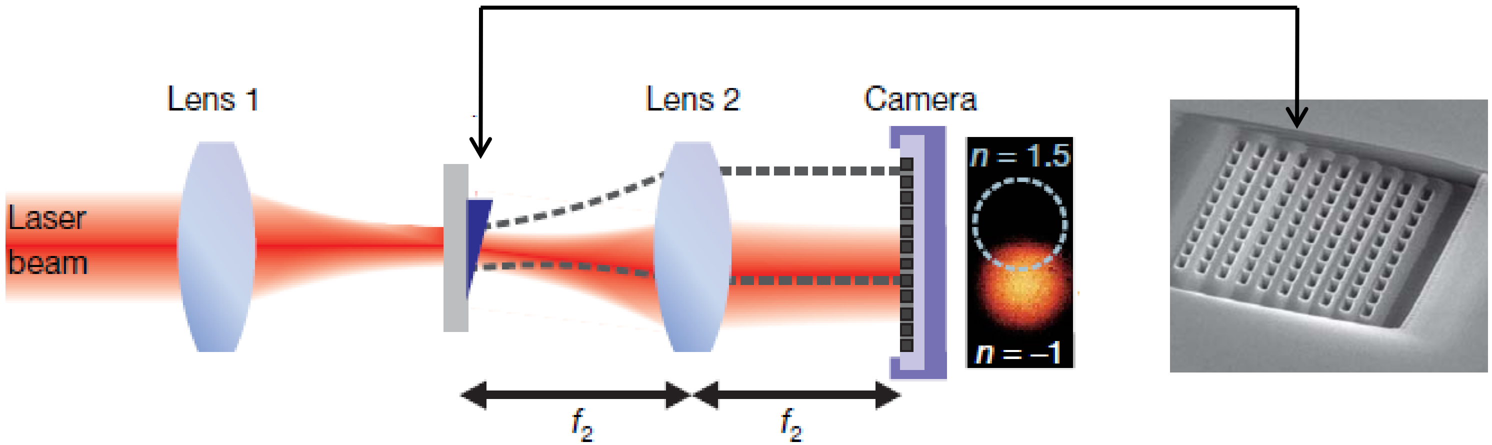


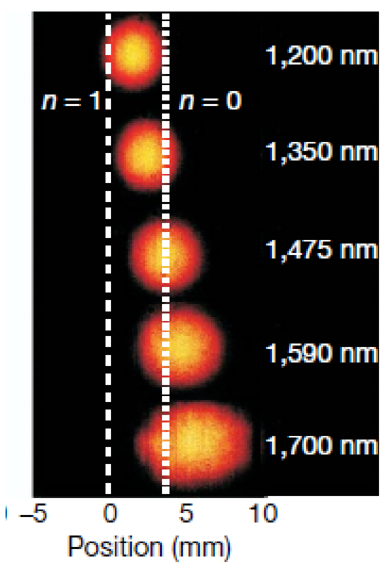

$p$ = 800 nm ~ $\lambda/2$

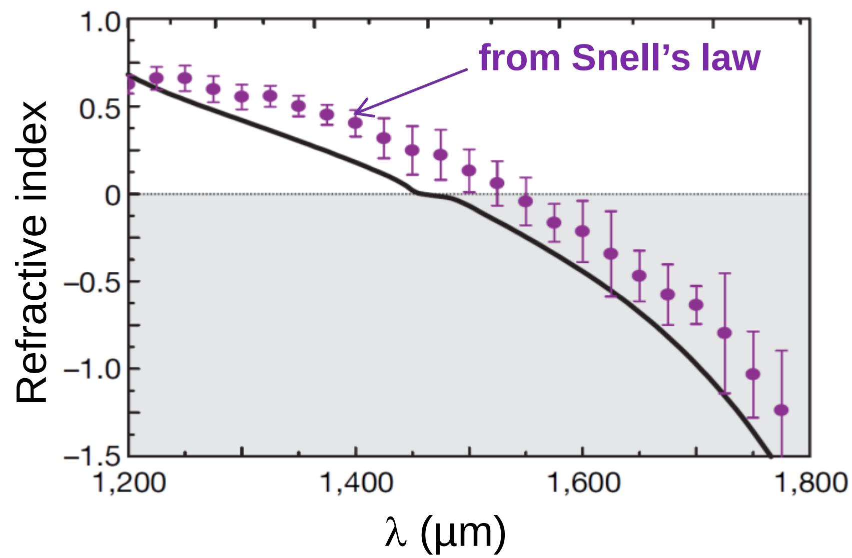



J. Valentine et al., Nature (London) **455**, 376 (2008)

# Optical negative refraction



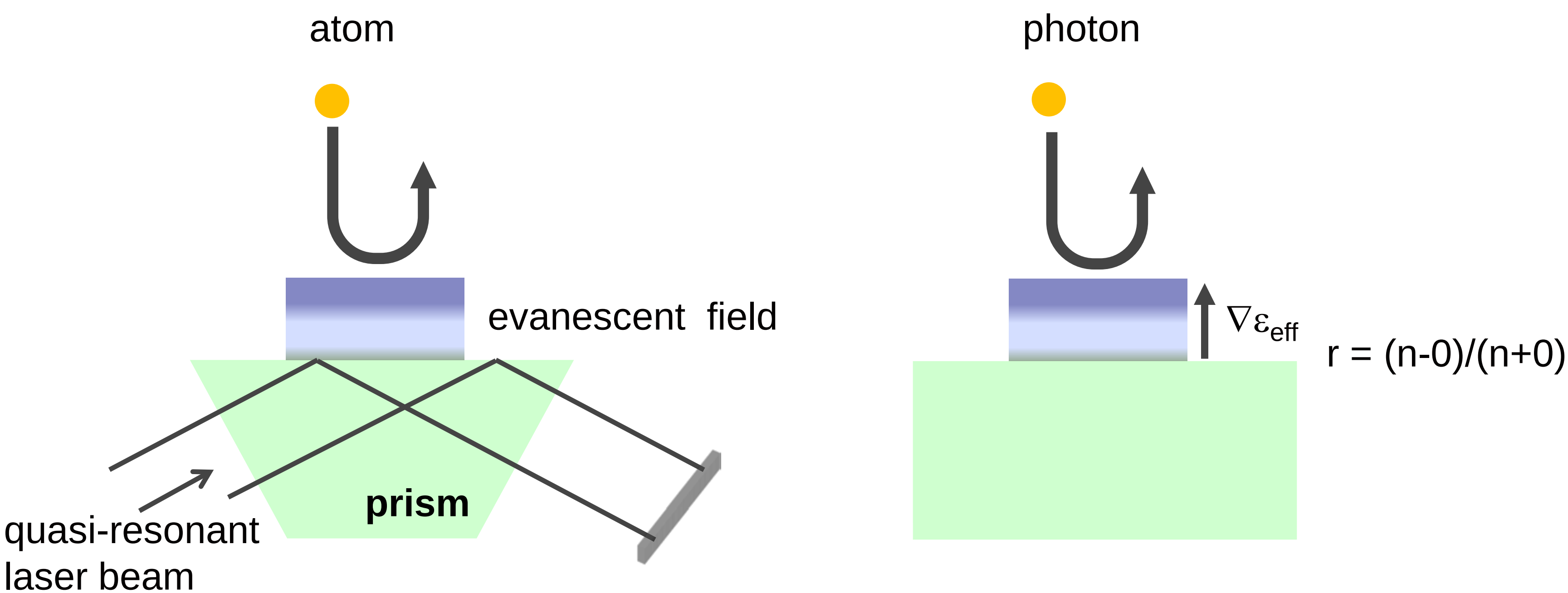




J. Valentine et al., Nature (London) **455**, 376 (2008)

# Conclusion

# Near-zero epsilons with atoms




Schrödinger ↔ Maxwell TE polarization

# Near-zero epsilons with atoms

Probability density $|\Psi|^2$

air

glass